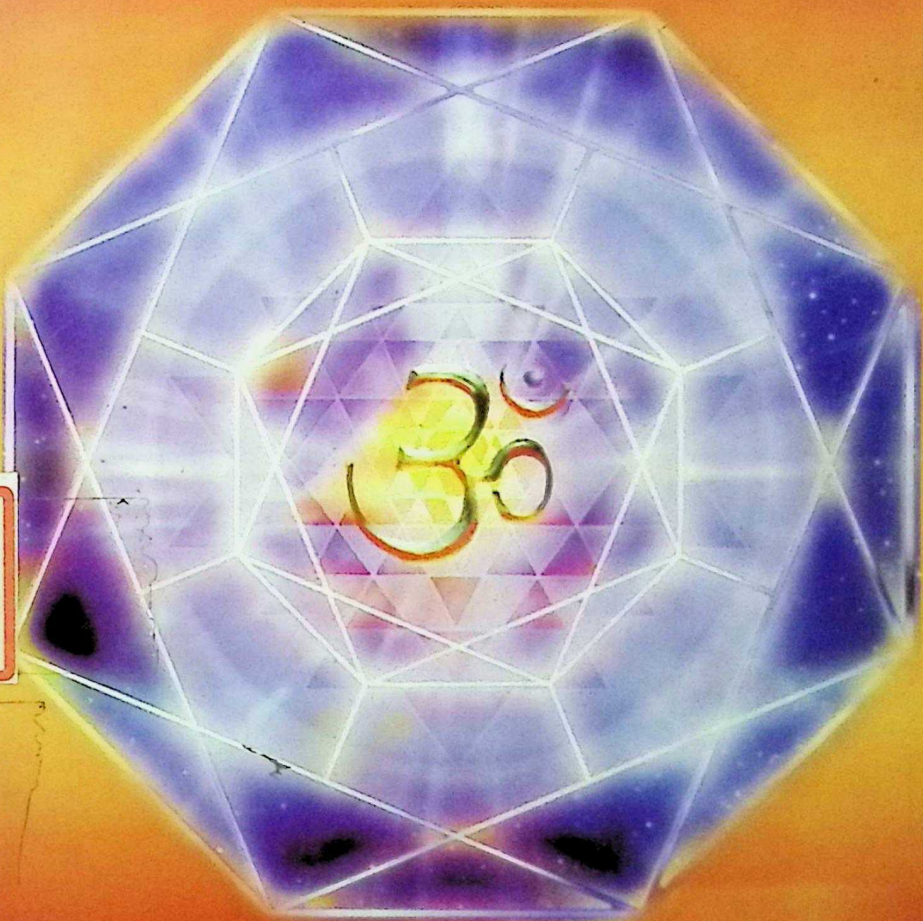


ENCYCLOPAEDIA OF VEDIC MATHEMATICS



SHYAM PRIYA



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Shyam Priya



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Preface

Vedic Mathematics contains many examples of striking methods of calculation and there is a remarkable coherence to the system which makes it very attractive. Also the Vedic system itself suggests a kind of approach that involves going directly to the answer. Vedic mathematics is based on sixteen Sutras and some sub-Sutras which provide links through mathematics: the word 'Sutra' means 'thread'.

Vedic scheme is not simply a collection of rapid methods; it is a system, a unified approach, which can be swiftly learnt. Vedic Mathematics extensively exploits the properties of numbers in every practical applications, particularly in the field of computation. It makes available a whole range of methods ideally suited to these properties. After a detailed introduction to Vedic Mathematics, this book shows applications of Vertically and Crosswise from basic calculations to the evaluation of logarithms, exponentials, trigonometrical functions and the solution of simultaneous, transcendental, polynomial and differential equations. The methods shown are not only exceedingly efficient and new but have a unity and coherence which renders them easy to master and enjoyable to execute.

The methods given in this book are not intended to be a complete or thorough treatment of the topics they deal with. All of the ideas can probably be developed further or applied in other areas, and all can doubtless be improved upon. The mathematician will also observe a certain lack of rigor as an attempt has been made to make the material intelligible to as wide a readership as possible.

—*Shyam Priya*

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Indian Mathematics: Redressing the Balance

Mathematics has long been considered an invention of European scholars, as a result of which the contributions of non-European countries have been severely neglected in histories of mathematics. Worse still, many key mathematical developments have been wrongly attributed to scholars of European origin. This has led to so-called Eurocentrism. The neglect of non-European mathematics is no more apparent than when studying the contributions of India. Contrary to Eurocentric belief, scholars from India, over a period of some 4500 years, contributed to some of the greatest mathematical achievements in the history of the subject. From the earliest numerate civilisation of the Indus valley, through the scholars of the 5th to 12th centuries who were conversant in arithmetic, algebra, trigonometry, geometry combinatorics and latterly differential calculus, Indian scholars led the world in the field of mathematics. The peak coming between the 14th and 16th centuries in the far South, where scholars were the first to derive infinite series expansions of trigonometric functions.

In addition to mighty contributions to all the principal areas of mathematics, Indian scholars were responsible for the creation, and refinement of the current decimal place value system of numeration, including the number zero,

without which higher mathematics would not be possible. The purpose of my project is to highlight the major mathematical contributions of Indian scholars and further to emphasise where neglect has occurred and hence elucidate why the Eurocentric ideal is an injustice and in some cases complete fabrication.

Introduction

The history of science, and specifically mathematics, is a vast topic and one which can never be completely studied as much of the work of ancient times remains undiscovered or has been lost through time.

Nevertheless there is much that is known and many important discoveries have been made, especially over the last 150 years, which have significantly altered the chronology of the history of mathematics, and the conceptions that had been commonly held prior to that. By the turn of the 21st century it was fair to say that there was definite knowledge of where and when a vast majority of the significant developments of mathematics occurred. However, despite widely available, reliable information, I became aware during the course of more general studies of the history of mathematics, that many discrepancies persist.

I became drawn to the topic of Indian mathematics, as there appeared to be a distinct and inequitable neglect of the contributions of the sub-continent. Thus, during the course of this project I aim to discuss that despite slowly changing attitudes there is still an ideology' which plagues much of the recorded history of the subject.

That is, to some extent very little has changed even in our seemingly enlightened historical and cultural position, and, in specific reference to my study area, many of the developments of Indian mathematics remain almost completely ignored, or worse, attributed to scholars of other nationalities, often European.

It is important for me to clarify at this point that the ideology I refer to is that held by (predominantly) European historians of science and mathematics, that mathematics is a European 'invention'. This ideology leads to an 'intrinsically' Eurocentric bias to the history of the subject. As R Rashed comments the ideology can be summarised as such:

...Classical science is European and its origins are directly traceable to Greek philosophy and science.
[RR, P 332]

Thus despite the many discoveries that have been made there has been a great reluctance to acknowledge the contributions of non-European countries.

For several hundred years following the European mathematical renaissance of the late 15th and early 16th century there was a commonly held opinion among commentators and historians that mathematics originated in its entirety from Europe and European scholars.

The basic chronology of the history of mathematics was very simple; it had primarily been the invention of the ancient Greeks, whose work had continued up to the middle of the first millenium A.D. Following which there was a period of almost 1000 years where no work of significance was carried out until the European renaissance, which coincided with the 'reawakening' of learning and culture in Europe following the so called dark ages.

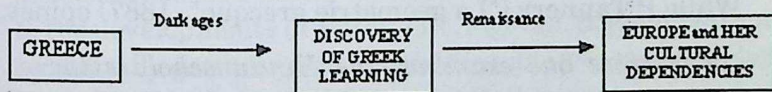


Figure : Eurocentric chronology of mathematics history.

Some historians made some concessions, by acknowledging the work of Egyptian, Babylonian, Indian and Arabic mathematicians (and occasionally the work of the Far East and China). Modified versions of the Eurocentric model commonly took the form seen below.

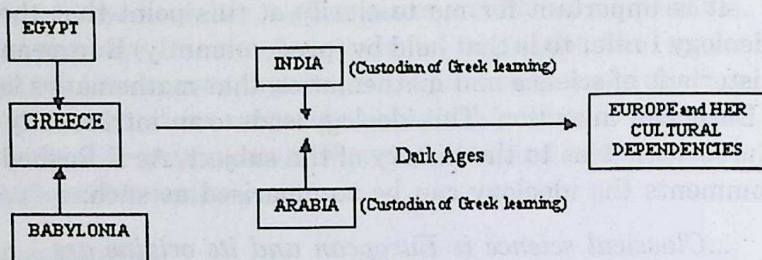


Figure : Modified Eurocentric model.

However, references to the work of these 'others' or 'non-Europeans' were always brief and hazy, and generally concluded that they were merely reconstructions of Greek works and that nothing of significance or importance was contained in them. Indian scholars, on the relatively rare occasions they were discussed, were merely considered to be custodians of Ancient Greek learning. There is a plethora of works and quotes that highlight the attitude that prevailed towards the works of so called non-European mathematics. P Duhem ("Le systeme du monde", 1965) states very simply:

...Arabic Science only reproduced the teachings received from Greek science. [RR, P 338]

Furthermore G Sarton ("Guide to the History of Science", 1927) comments:

...One could almost omit Hindu and Chinese developments in Mathematics. [AA'D, P 15]

While P Tannery ("La geometrie grecque", 1887) opines:

...The more one examines the Hindu scholars the more they appear dependent upon the Greeks... (and)...quite inferior to their predecessors on all respects. [RR, P 338]

It is this underhand and in some cases completely fallacious attitude towards Indian mathematics that I have chosen to focus upon. As said this is a vast topic area and although all non-European 'roots' of mathematics have

Indian Mathematics: Redressing the Balance

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suffered neglect and miss-representation by many historians I am not going to attempt to focus on all non-European roots of mathematical development. I have chosen to focus on the mathematical developments of the Indian sub-continent, as I consider them not only to be severely neglected in histories of mathematics, but also to have produced some of the most remarkable results of mathematics. Indeed, the research I have conducted has highlighted that many Indian mathematical results, beyond being simply remarkable because of the time in which they were derived, show that several 'key' mathematical topics, and subsequent results, indubitably originate from the Indian subcontinent. Having made these discoveries it seems to me an incredible injustice that the work of Indian scholars is not rewarded with a much more prominent place in the history of mathematics.

Not only have many historians identified Greek influences in all of the work of Indian scholars, (in some cases by suggesting severely inaccurate dates for the works), several others have even attempted to show Arabic influences, which is quite incredible as several Indian works in fact had a significant influence on early Arab works. Although I by no means wish to talk down the incredible developments of Arab scholars, I believe the developments of Indian scholars are on a par with, and occasionally surpass them.

Indian scholars made vast contributions to the field of mathematical astronomy and as a result contributed mightily to the developments of arithmetic, algebra, trigonometry and secondarily geometry (although this topic was well developed by the Greeks) and combinatorics. Perhaps most remarkable were developments in the fields of infinite series expansions of trigonometric expressions and differential calculus.

Surpassing all these achievements however was the development of decimal numeration and the place value system, which without doubt stand together as the most remarkable developments in the history of mathematics,

and possibly one of the foremost developments in the history of humankind. The decimal place value system allowed the subject of mathematics to be developed in ways that simply would not have been possible otherwise. It also allowed numbers to be used more extensively and by vastly more people than ever before.

The aim of my work is not just to paint as accurate a picture of the developments of mathematics by the Indian peoples as possible, but also to attempt to give reasons as to why Europe has chosen to neglect the facts of history. Through a detailed discussion of the mathematics of the Indian subcontinent I hope to highlight why this 'neglect' is such an injustice, and briefly discuss the possible consequences of this neglect.

There are several points that I feel it is important to make before progressing further with my discussion. The first is to make it clear that the chronology of the history of mathematics is not entirely linear. One will often find simplified diagrammatic representations where the work of one group of people (or country) is proceeded by the work of another group and so on. In reality things are far more complicated than this. Particularly in the European dark ages (5th-15th centuries) mathematical developments passed between several countries, being constantly refined and improved. G Joseph states:

... A variety of mathematical activity and exchange between a number of cultural areas went on while Europe was in a deep slumber. [GJ, P 9]

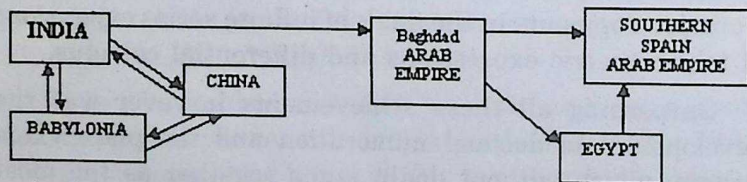


Figure : Non-European mathematics during the dark ages.

Secondly it is worth noting that we view the history of mathematics from:

...Our own position of understanding and sophistication. [EFR/JO'C1, P 3]

While this is unavoidable it is vital to appreciate the uniqueness and ingenuity of the developments of Indian scholars, even if the results are common-place now.

We also view mathematics from an intrinsically European standpoint due largely to the influence of European scholars over the last 500 years and the colonisation of much of the world by European countries. Perhaps this is why many historians find it hard to accept that many results and developments of mathematics are not European in origin. In short, if we are European, somewhat unavoidably we view history from our indigenous standpoint.

There is also a slight methodological problem related to the 'labeling' of the topic of "Indian mathematics" that I will briefly discuss. Using the label "Indian mathematics" is not entirely accurate as much of the earliest "Indian mathematics" was developed in areas, which are now part of Pakistan. The label is used for simplicity and can be justified by stating that developments took place in the Indian subcontinent. Quite often the 'label' Hindu mathematics is used, but it is less accurate as many of the scholars were not Hindus.

Finally before progressing I will specify the time period that my work will cover. The earliest origins of Indian mathematics have been dated to around 3000 BC and this seems a sensible point at which to commence my discussion, while work of a significant nature was still being carried out in the south of India in the 16th century, following which there was an eventual decline. It is hence a vast time scale of almost 5000 years, and indeed it may be greater than that, the estimation of 3000 BC is a slightly crude

approximation, and there remains much controversy with regards to the dating of many works prior to 400 AD.

As Gupta states in his paper on the problem of ancient Indian chronology:

...In the case of India, the problem of chronology continues to be very serious especially with regard to the prehistoric and ancient periods. [RG1, P 17]

It is also worth pointing out that this lack of certainty has allowed several unscrupulous scholars to pick dates of choice for certain Indian discoveries so as to justify suggestions for Greek, Arab or other influences. In situations where this has arisen I will attempt to the best of my ability to state fact, although in some cases a well-informed guess will have to suffice.

Early Indian Culture—Indus Civilisation

The first appearance of evidence of the use of mathematics in the Indian subcontinent was in the Indus valley (see Figure) and dates back to at least 3000 BC. Excavations at Mohenjodaro and Harrapa, and the surrounding area of the Indus River, have uncovered much evidence of the use of basic mathematics. The maths used by this early Harrapan civilisation was very much for practical means, and was primarily concerned with weights, measuring scales and a surprisingly advanced 'brick technology', (which utilised ratios). The ratio for brick dimensions 4:2:1 is even today considered optimal for effective bonding.

The discoveries of systems of uniform and decimal weights, over a vast area, are of considerable interest. G Joseph states:

...Such standardisation and durability is a strong indication of a numerate culture. [GJ, P 222]

Also, many of the weights uncovered have been produced in definite geometrical shapes (cuboid, barrel, cone, and

cylinder to name a few) which present knowledge of basic geometry, including the circle.

This culture also produced artistic designs of a mathematical nature and there is evidence on carvings that these people could draw concentric and intersecting circles and triangles, leading S Sinha to state:

...The civilisation and culture of the inhabitants of the Indus valley...were of a very advanced nature.
[SS1, P 71]

S Srinivasan further comments:

...There are many unique features in the construction patterns, which suggest an independent origin of ideas in ancient Indian civilisation. [SSr1, P17]

Further to the use of circles in 'decorative' design there is indication of the use of bullock carts, the wheels of which may have had a metallic band wrapped round the rim. This clearly points to the possession of knowledge of the ratio of the length of the circumference of the circle and its diameter, and thus values of π .

Also of great interest is a remarkably accurate decimal ruler known as the Mohenjodaro ruler. Subdivisions on the ruler have a maximum error of just 0.005 inches and, at a length of 1.32 inches, have been named the Indus inch. Furthermore, a correspondence has been noted between the Indus scale and brick size. Bricks (found in various locations) were found to have dimensions that were integral multiples of the graduations of their respective scales, which suggests advanced mathematical thinking.



Figure : Ruler found at Lothal. [SSr1, P17]

Above all else there are also brief references to an early decimal system of numeration. The seeds of what were to become the single greatest contribution of the Indian sub-continent to the world (not just of mathematics) had already been sown. My evidence comes from S Sinha who states:

...Writers on these civilisations briefly refer to the decimal system of numeration found in these excavations. [SS1, P 71]

This quote supports the theory that the Brahmi numerals, which were to go on to develop into the numerals we use today, originated in the Indus valley around 2000 BC, however this theory has been rejected by several scholars including Ifrah and Joseph. This quote could be considered a piece of overzealous reporting by the author however, on further investigation I can support the comment with some confidence.

Not only are the markings on all the excavated measuring devices decimal in nature, but there is also research currently being conducted, which is attempting, with success, to show a connection between the Brahmi and Indus scripts. This lends indirect support to suggestions of the existence of early decimal numeral forms. As I will discuss briefly later, the Brahmi numerals undoubtedly developed into the numeral forms we use today.

Although this early mathematics is generally included in histories of mathematics it is often in nothing more than a brief mention, and there is a most curious quote by J Katz who claims:

...There is no direct evidence of its (Harappan civilisation) mathematics. [JK, P4]

It is possible that he makes this comment with regards to the fact that the Indus script as yet remains undeciphered (GJ, P218).

However R Gupta more 'sensibly' states:

...In fact the level of mathematical knowledge implied in various geometrical designs, accurate layout of streets and drains and various building constructions etc. was quite high (from a practical point of view). [RG1, P131]

While Childe claims:

...India confronts Egypt and Babylonia by the 3rd millennium with a thoroughly individual and independent civilisation of her own. [EFR/JJO'C2, P 1]

Some confusion exists as to what caused the decline of this Harappan culture, there are several theories, the most probable of which in my opinion was the drying up of the Sarasvati River.

This view is supported by S Kak and also S Kalyanaraman who has written an extensive paper on the topic and comments:

... The drying-up of the Sarasvati River led to migrations of people eastwards.

The most commonly held view by historians is that Aryan peoples from the North invaded and destroyed the Harappan culture, this view however is considered increasingly contentious.

In addition to the significance the fledgling decimal system would ultimately have, the most important legacy of this early civilisation is the influence its brick technology may have had on the altar building required by the Vedic religion that followed. A theory of the 'interlinkage' of the Harappan and Vedic cultures has recently arisen from a variety of studies, and it may come to light that there was a greater interaction between the two civilisations than currently thought.

Mathematics in the Service of Religion: Vedas and Vedangas

The Vedic religion was followed by the Indo-Aryan peoples, who originated from the north of the sub-continent. It is through the works of Vedic religion that we gain the first literary evidence of Indian culture and hence mathematics. Written in Vedic Sanskrit the Vedic works, Vedas and Vedangas (and later Sulbasutras) are primarily religious in content, but embody a large amount of astronomical knowledge and hence a significant knowledge of mathematics. The requirement for mathematics was (at least at first) twofold, as R Gupta discusses:

...The need to determine the correct times for Vedic ceremonies and the accurate construction of altars led to the development of astronomy and geometry.
[RG2, P 131]

Some chronological confusion exists with regards to the appearance of the Vedic religion. S Kak states in a very recent work that the time period for the Vedic religion stretches back potentially as far as 8000BC and definitely 4000BC. Whereas G Joseph states 1500 BC as the forming of the Hindu civilisation and the recording of Vedas and Vedangas, and later Sulbasutras. However it seems most likely that significant knowledge of astronomy and mathematics first appears in Vedic works around the 2nd millennium BC. The Rg-Veda (fire altar) the earliest extant Vedic work dates from around 1900 BC. R Gupta in his paper on the problem of ancient Indian chronology shows that dates from 26000-200 BC have been suggested for the Vedic 'period'. Having consulted many sources I am confident at placing the period of the Vedas (and Vedangas) at around 1900-1000 BC.

Further mathematical work is found in the Sulbasutras of the later Vedic period, the earliest of which is thought to have been written around 800 BC and the last around

200 BC. I will now move on from this slightly clouded chronological discussion. It is however worth noting that there are serious underlying problems with the chronology of early Indian mathematics which require significant attention.

Although the requirement of mathematics at this time was clearly not for its own sake, but for the purposes of religion and astronomy, it is important not to ignore the secular use of the texts, i.e. by the craftsmen who were building the altars.

Similarly with the earlier Harappan peoples it seems likely that (at least) basic mathematics will have grown to become used by large numbers of the population. Regardless of the fact that at this time mathematics remained for practical uses, some significant work in the fields of geometry and arithmetic were developed during the Vedic period and as L Gurjar states:

...The Hindu had made enormous strides in the field of mathematics. [LG, P 2]

It is also worthwhile briefly noting the astronomy of the Vedic period which, given very basic measuring devices (in many cases just the naked eye), gave surprisingly accurate values for various astronomical quantities. These include the relative size of the planets the distance of the earth from the sun, the length of the day, and the length of the year. For further information, S Kak is an authority on the astronomical content of Vedic works.

Much of the mathematics contained within the Vedas is found in works called Vedangas of which there are six. Of the six Vedangas those of particular significance are the Vedangas Jyotis and Kalpa (the fifth and sixth Vedangas). Jyotis was (at the time) the name for astronomy, while Kalpa contained the rules for the rituals and ceremonies. The Vedangas are best described as an auxiliary to the Vedas.

N Dwary claims, with reference to the Vedanga-Jyotis, that:

...Hindus of the period were fully conversant with fundamental operations of arithmetic. [ND1, P 39]

S Kak suggests a date of around 1350 BC for the Vedanga-Jyotis. I include this as a reminder of the time period being discussed.

Along with the Vedangas there are several further works that contain mathematics, including:

Taittiriya Samhita

Satapatha Brahmana and

Yajur and Atharva-Veda

Rigveda (of which it is thought there are three 'versions') plus additional

Samhitas

Of these the Taittiriya Samhita and Rg-Veda are considered the oldest and contain rules for the construction of great fire altars.

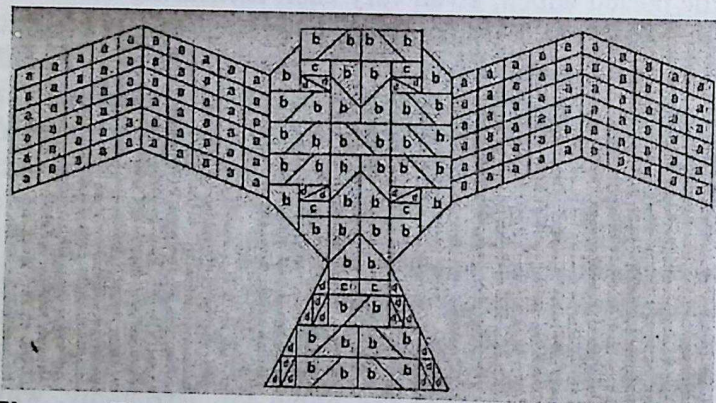


Figure : First layer of a Vedic sacrificial altar (in the shape of a falcon). [GJ, P 227]

As a result of the mathematics required for the construction of these altars, many rules and developments

of geometry are found in Vedic works. These include: Use of geometric shapes, including triangles, rectangles, squares, trapezia and circles.

Equivalence through numbers and area.

Equivalence led to the problem of:

Squaring the circle and visa-versa.

Early forms of Pythagoras theorem.

Estimations for π .

S Kak gives three values for from the Satapatha Brahmana. It seems most probable that they arose from transformations of squares into circles and circles to squares. The values are:

$$\pi_1 = \frac{25}{8} (3.125)$$

$$\pi_2 = \frac{900}{289} (3.11418685...)$$

$$\pi_3 = \frac{1156}{361} (3.202216...)$$

Astronomical calculations also leads to a further Vedic approximation:

$$\pi_4 = \frac{339}{108} (3.1389)$$

This is correct (when rounded) to 2 decimal places.

Also found in Vedic works are:

All four arithmetical operators (addition, subtraction, multiplication and division).

A definite system for denoting any number up to 1055 and existence of zero.

Prime numbers.

The Arab scholar Al-Biruni (973-1084 AD) discovered that only the Indians had a number system that was capable of going beyond the thousands in naming the orders in decimal counting.

Evidence of the use of this advanced numerical concept leads S Sinha to comment:

...It is fair to agree that a nation with such an advanced and cultured civilisation and which was using the numerical system (decimal place value) knew also how to handle the associated arithmetic. [SS1, P 73]

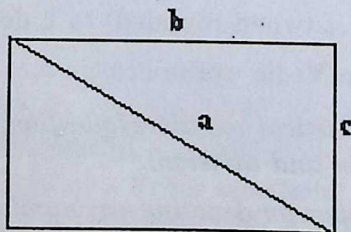
It is in Vedic works that we also first find the term "ganita" which literally means "the science of calculation".

It is basically the Indian equivalent of the word mathematics and the term occurs throughout Vedic texts and in all later Indian literature with mathematical content.

Among the other works I have mentioned, mathematical material of considerable interest is found:

Arithmetical sequences, the decreasing sequence 99, 88, ... , 11 is found in the Atharva-Veda.

Pythagoras's theorem, geometric, constructional, algebraic and computational aspects known. A rule found in the Satapatha Brahmana gives a rule, which implies knowledge of the Pythagorean theorem, and similar implications are found in the Taittiriya Samhita.



Fractions, found in one (or more) of the Samhitas.

Equations, $972 \times 2 = 972 + m$ for example, found in one of the Samhitas.

The 'rule of three'.

Sulba Sutras

The later Sulba-sutras represent the 'traditional' material along with further related elaboration of Vedic mathematics. The Sulba-sutras have been dated from around 800-200 BC, and further to the expansion of topics in the Vedangas, contain a number of significant developments.

These include first 'use' of irrational numbers, quadratic equations of the form $ax^2 = c$ and $ax^2 + bx = c$, unarguable evidence of the use of Pythagoras theorem and Pythagorean triples, predating Pythagoras (c 572 - 497 BC), and evidence of a number of geometrical proofs. This is of great interest as proof is a concept thought to be completely lacking in Indian mathematics.

EXAMPLE 1:

Pythagoras theorem and Pythagorean triples, as found in the Sulba Sutras.

The rope stretched along the length of the diagonal of a rectangle makes an area which the, vertical and horizontal sides make together.

In other words:

$$a^2 = b^2 + c^2$$

Examples of Pythagorean triples given as the sides of right angled triangles:

5, 12, 13

8, 15, 17

12, 16, 20

12, 35, 37

Of the Sulvas so far 'uncovered' the four major and most mathematically significant are those composed by Baudhayana, Manava, Apastamba and Katyayana (perhaps least 'important' of the Sutras, by the time it was composed the Vedic religion was becoming less predominant).

However in a paper written 20 years ago S Sinha claims that there are a further three Sutras, 'composed' by Maitrayana, Varaha and Vadhula (SS1, P 76). I have yet to come across any other references to these three 'extra' sutras.

These men were not mathematicians in the modern sense but they are significant none the less in that they were the first mentioned 'individual' composers. E Robertson and J O'Connor have suggested that they were Vedic priests (and skilled craftsmen).

It is thought that the Sulvas were intended to supplement the Kalpa (the sixth Vedanga), and their primary content remained instructions for the construction of sacrificial altars. The name Sulvasutra means 'rule of chords' which is another name for geometry.

N Dwary states:

...They offer a wealth of geometrical as well as arithmetical results. [ND1, P 40]

R Gupta similarly claims:

...The Sulba-sutras are (quite) rich in mathematical contents. [RG2, P 133]

With reference to the possible appearance of proof is a quote from A Michaels:

...Vedic geometry, though non-axiomatic in character, is provable and indeed proof is implicit in several constructions prescribed in the Sulba-sutras. [RG2, P 133]

This is not particularly compelling evidence but does suggest that the composers of the sulba-sutras may have had a greater depth of knowledge than is generally thought.

Many suggestions for the value of $\sqrt{2}$ are found within the sutras. They cover a surprisingly wide range of values, from 2.99 to 3.2022.

Pythagoras's theorem and Pythagorean triples arose as the result of geometric rules. It is first found in the Baudhayana sutra - so was hence known from around 800 BC. It is also implied in the later work of Apastamba, and Pythagorean triples are found in his rules for altar construction. Altar construction also led to the discovery of irrational numbers, a remarkable estimation of $\sqrt{2}$ is found in three of the sutras. The method for approximating the value of $\sqrt{2}$ gives the following result:

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.434}$$

This is equal to 1.412156..., which is correct to 5 decimal places.

It has been argued by scholars seemingly attempting to deprive Indian mathematics of due credit, that Indians

believed that $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3.4} - \frac{1}{3.434}$ exactly, which would not indicate knowledge of the concept of irrationality. Elsewhere in Indian works however it is stated that various square root values cannot be exactly determined, which strongly suggests an initial knowledge of irrationality.

Indeed an early method for calculating square roots can be found in some Sutras, the method involves repeated application of the formula: $\sqrt{A} = \sqrt{(a^2 + r)} = a + \frac{r}{2a}$, r being small.

EXAMPLE 2:**Application of formula for calculating square roots.**

If $A = 10$, take $a = 9$ and $r = 1$.

Thus $\sqrt{10} = \sqrt{(32+1)} = a + \frac{r}{2a} = 3 + \frac{1}{6} = 3.16667$ in (modern) decimal notation.

$\sqrt{10} = 3.162278$ to six decimal places when calculated on a calculator. Thus, after only one application of the formula, a moderately accurate value has been calculated.

C Srinivasiengar thus states:

...The credit of using irrational numbers for the first time must go to the Indians. [CS, P 15]

Many of the Vedic contributions to mathematics have been neglected or worse. When it first became apparent that there was geometry contained within works that was not of Greek origin, historians and mathematical commentators went to great lengths to try and claim that this geometry was Greek influenced (to a greater or lesser extent).

It is undeniable that none of the methods of Greek geometry are discernible in Vedic geometry, but this merely serves to support arguments that it is independently developed and not in some way borrowed from Greek sources.

In light of recent evidence and more accurate dating it has been even more strongly claimed by A Seidenberg (in S Kak) that:

...Indian geometry and mathematics pre-dates Babylonian and Greek mathematics. [SK1, P 338]

This may be a somewhat extreme standpoint, and it seems likely that there was traffic of ideas in all directions of the Ancient world, but there is little doubt that the vast majority of Indian work is original to its writers. It may lack the cold logic and truly abstract character of modern

mathematics but this observation further helps to identify it as uniquely Indian. Of all the mathematics contained in the Vedangas it is the definite appearance of decimal symbols for numerals and a place value system that should perhaps be considered the most phenomenal.

Before the period of the Sulbasutras was at an end the Brahmi numerals had definitely begun to appear (c. 300BC) and the similarity with modern day numerals is clear to see (see Figures).

More importantly even still was the development of the concept of decimal place value. M Pandit in a recent paper (discussed in RG2) has shown certain rules given by the famous Indian grammarian Panini (c. 500 BC) imply the concept of the mathematical zero. Further to this there is a small amount of evidence of the use of symbols for numbers even earlier in the Harrapan culture. My evidence comes primarily from a paper by S Kak, which analyses some of Panini's work, and there is further support from a paper by S Sinha. B Datta and A Singh also give evidence of an early emergence of numerical forms and the decimal place value system.

Jainism

Following the decline of the Vedic religion around 400BC, the Jaina religion (and Buddhism) became the prominent religion(s) on the Indian subcontinent and gave rise to Jaina mathematics. N Dwary (and others) contend:

...According to the religious literature of the Janias, the knowledge of "Sankhyana" (i.e., the science of numbers, which included arithmetic and astronomy) was considered to be one of the principal accomplishments of Jain priests. [ND1, P 39]

The main Jaina works on mathematics date from around 300BC to 400AD, but the Jaina religion was in its infancy

as far back as 500BC. Further, as a point of slight interest, the Jaina religion has still not completely died out, and up till the 19th century some very minor works were produced.

Jaina mathematics played an important role in bridging the gap between 'ancient' Indian mathematics and the so-called 'Classical Period', which was heralded by the work of Aryabhata I in the 6th century.

Regrettably there are few extant Jaina works, but in the limited material that exists an incredible level of originality is demonstrated. Perhaps the most historically important Jaina contribution to mathematics as a subject is the progression of the subject from purely practical or religious requirements. During the Jaina period mathematics became an abstract discipline to be cultivated "for its own sake". G Joseph states:

...The Jaina contribution to this change should be recognised. [GJ, P 250]

Despite its important historical position, relatively little attention has been paid to Jaina mathematics, and it remains seemingly discarded by many historians. This however is a massive oversight, because within Jaina works there are many remarkable 'new' results, and developments of topics found in Vedic works. It is worth briefly noting here that there is great uncertainty as to whether Vedic works influenced the Jaina mathematicians.

Throughout the history of Indian mathematics it seems very possible each 'leap' was made without knowledge of previous discoveries. This has been suggested to be primarily due to the size of the sub-continent and imperfect channels of communication within it. Another contribution was the 'oral transmission' tradition of Ancient Indian mathematics, which resulted in knowledge being lost over time.

There are several significant Jaina works, including the Surya Prajinapti (4th c. BC) and several Sutras. There is

also evidence of individual mathematicians including Bhadrabahu (c. 300 BC, possibly lived in the Mysore State), C Srinivasiengar conjectures he wrote two works (which have yet to be unearthed), and Umaswati, a commentator from 2nd century BC, (possibly lived around 150 BC).

Umaswati is known as a great writer on Jaina metaphysics but also wrote a work *Tattvarthadigama-Sutra Bhashya*, which contains mathematics.

Among the important developments of the Jainas are:

Theory of numbers.

There is a great fascination in Jain philosophy with the enumeration of large numbers, selected examples of 'time periods' mentioned include 756 1011 8400000028 days, called shirsa prahelika, and 2588 years.

All numbers were classified into three sets:

Enumerable, Innumerable and Infinite.

Five different types of infinity are recognised in Jaina works:

Infinite in one and two directions, infinite in area, infinite everywhere and infinite perpetually.

This theory is quite incredible and was not realised in Europe until the late 19th century work of George Cantor. Indeed much of the Jaina theory of infinity is extremely advanced for the time in which it was conceived.

Also found in Jaina works:

Knowledge of the fundamental laws of indices.

Arithmetical operations.

Geometry.

Operations with fractions.

Simple equations.

Cubic equations.

Quartic equations (the Jaina contribution to algebra is severely neglected).

Formula for (root 10, comes up almost inadvertently in a problem about infinity).

Operations with logarithms.

Sequences and progressions.

Finally of interest is the appearance of Permutations and Combinations in Jaina works, which resulted in the formation of an early Pascal triangle, called Meru Prastara, many centuries before Pascal himself 'invented' it. This is another case where Indian contributions have been neglected severely.

EXAMPLE 1:

Meru Prastara rule found in Jain works.

Rule is simpler than that of Pascal, and is based on the simple formula:

$${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$$

EXAMPLE 2:

Formulas for permutations and combinations.

Correct formulas for both permutations and combinations are found in Jaina works:

$${}_nC_1 = n, {}_nC_2 = n(n-1)/1.2, {}_nC_3 = n(n-1)(n-2)/1.2.3$$

$${}_nP_1 = n, {}nP_2 = n(n-1), {}nP_3 = n(n-1)(n-2)$$

The contribution of the Buddhist school should also be briefly discussed. Although in the shadow of Jaina developments, evidence suggests Buddhist scholars were well versed in the use of the decimal place values system and that knowledge of Gainta was considered important.

There was no sudden decline of the Jaina religion as such but from the beginning of the 6th century the work of

a mathematician named Aryabhatta surpassed all previous work of the Indian sub-continent and brought about the 'Classical period' of Indian mathematics, (which lasted 600 years). However prior to discussing the work of Aryabhatta there is a major piece of Indian mathematical work yet to be discussed. That is the Bakhshali manuscript.

The Bakhshali Manuscript

The Bakhshali manuscript, which was unearthed in the 19th century, does not appear to belong to any specific period. Although that said, G Joseph classes it as a work of the early 'classical period', while E Robertson and J O'Connor suggest it may be a work of Jaina mathematics, and while this is chronologically plausible there is no proof it was composed by Jain scholars. L Gurjar discusses its date in detail, and concludes it can be dated no more accurately than 'between 2nd century BC and 2nd century AD'. He offers compelling evidence by way of detailed analysis of the contents of the manuscript (originally carried by R Hoernle).

His evidence includes the language in which it was written ('died out' around 300 AD), discussion of currency found in several problems, and the absence of techniques known to have been developed by the 5th century. Further support of these dates is provided by several occurrences of terminology found only in the manuscript, (which form the basis of a paper by M Channabasappa).

The controversy and debate surrounding the date of the Bakhshali manuscript was particularly intense when it was first discovered and highlights the resistance of European historians to accept new discoveries and evidence of the origins of various mathematical results. Clearly establishing a date for the composition of the manuscript is extremely important as it has a vast bearing on the significance of its mathematical content. This is in fact the primary requirement for accurate dating of all mathematical works.

The first translation of the manuscript was carried out by G R Kaye, however he was quite unscrupulous in his work and attempted to date the manuscript as 12th Century in order to justify Arabic and Greek influences on the text. Kaye even went as far as to question the Indian origin of the manuscript.

However the vast majority of his translation and suggestions made about the origins of the manuscript have been debunked by less biased (and more accurate) translations. G Joseph further criticises Kaye and comments:

...It is particularly unfortunate that Kaye is still quoted as an authority on Indian mathematics. [GJ, Ps 215-216]

To slightly confuse the issue, it is now considered (almost without doubt) that the manuscript found at Bakhshali is a copy (of the original work) dating from around the 8th century, and certainly no later than 950 AD. The scholar R Hoernle was the first to reach this conclusion following detailed analysis of the manuscript.

I am content to agree with the (prominent) historians who have placed the date at pre 450 AD and identified the 'current' version as a copy. Avoiding further debate, L Gurjar states that the Bakshali manuscript is the:

...Capstone of the advance of mathematics from the Vedic age up to that period. [LG, P49]

Although, as much work was lost between 'periods', we cannot fully gauge continuity of progress and it is possible the composer(s) of the Bakhshali manuscript were not fully aware of earlier works and had to start from 'scratch'. This would make the work an even more remarkable achievement.

The B. Ms. was written on leaves of birch, in Sarada characters and in Gatha dialect, which is a combination of Sanskrit and Prakrit. This may go some way to explaining the number of inaccurate translations. Many of the historians

who have been involved in translating ancient Indian works have done so poorly, due to the obscure script, or alternatively because they have not understood the mathematics fully. More worryingly there could be unscrupulous reasons for poor translating in order to play down the importance of ancient Indian works, because they challenge the Eurocentric ideal.

The B. Ms. highlights developments in Arithmetic and Algebra. The arithmetic contained within the work is of such a high quality that it has been suggested:

...In fact [the] Greeks [are] indebted to India for much of the developments in Arithmetic. [LG, P 53]

This quote 'throws open' the traditional Eurosceptic opinion of the history (and origins) of mathematics. Yet even today histories of mathematics rarely acknowledge this contribution of the Indian sub-continent and the B. Ms. is rarely if ever mentioned.

There are eight principal topics 'discussed' in the B. Ms:

Examples of the rule of three (and profit and loss and interest).

Solution of linear equations with as many as five unknowns.

The solution of the quadratic equation (development of remarkable quality).

Arithmetic (and geometric) progressions.

Compound Series (some evidence that work begun by Jainas continued).

Quadratic indeterminate equations (origin of type $ax/c = y$).

Simultaneous equations.

Fractions and other advances in notation including use of zero and negative sign.

Improved method for calculating square root (and hence approximations for irrational numbers). The improved method (shown below) allowed extremely accurate approximations to be calculated:

$$\sqrt{A} = \sqrt{a^2 + r} = a + \frac{r}{2a} - \left\{ \left(\frac{r}{2a} \right)^2 / 2 \left(\frac{a+r}{2a} \right) \right\}$$

EXAMPLE 1:

Application of square root formula.

Again we can calculate $\sqrt{10}$, where $a = 3$ and $r = 1$.

$$\begin{aligned} \sqrt{10} &= \sqrt{(3^2 + 1)} = 3 + \frac{1}{6} - \left\{ (1/36) / 2(3 + 1/6) \right\} \\ &= 3 + 1/6 - \{ (1/36) / (19/3) \} \\ &= 3 + 1/6 - 1/228 \\ &= 3.16228... \text{ in decimal form} \end{aligned}$$

$\sqrt{10} = 3.16228$ when calculated on a calculator and rounded to five decimal places.

EXAMPLE 2:

Quadratic equation as found in B. Ms.

If the equation given is $dn^2 + (2a - d)n - 2s = 0$

Then the solution is found using the equation:

$$n = \frac{-(2a - d) \pm \sqrt{(2a - d)^2 + 8ds}}{2d}$$

Which is the quadratic equation with $a = d$, $b = 2a - d$, and $c = 2s$.

EXAMPLE 3:

Linear equation with 5 variables.

The following problem is stated in the B. Ms:

"Five merchants together buy a jewel. Its price is equal to half the money possessed by the first together with the

money possessed by the others, or one-third the money possessed by the second together with the moneys of the others, or one-fourth the money possessed by the third together with the moneys of the others...etc. Find the price of the jewel and the money possessed by each merchant.

SOLUTION

We have the following systems of equations:

$$x_{1/2} + x_2 + x_3 + x_4 + x_5 = p$$

$$x_1 + x_{2/3} + x_3 + x_4 + x_5 = p$$

$$x_1 + x_2 + x_{3/4} + x_4 + x_5 = p$$

$$x_1 + x_2 + x_3 + x_{4/5} + x_5 = p$$

$$x_1 + x_2 + x_3 + x_4 + x_{5/6} = p$$

Then if $x_{1/2} + x_{2/3} + x_{3/4} + x_{4/5} + x_{5/6} = q$ the equations become $(\frac{377}{60})q = p$.

A number of possible answers can be obtained. This is the origin of the indeterminate equation of the type $ax/c = y$, the theory of which was greatly developed, and later perfected by Bhaskara II, four hundred years before it was discovered in Europe.

If $q = 60$ then $p = 377$ and $x_1 = 120$, $x_2 = 90$, $x_3 = 80$, $x_4 = 75$ and $x_5 = 72$

With regard to my central discussion of the neglect of Indian mathematics I feel it is important to highlight the presence of moderately advanced algebra in the B. Ms. Historians of mathematics debate whether true algebra 'began' in Greece or Arabia, and little mention is ever made of Indian algebra. In light of my own research I feel that early Arabic algebra (c. 800 AD) in no way surpasses the level of understanding of 6th century Indian scholars.

The Bakhshali manuscript is a unique piece of work and while it not only contains mathematics of a remarkably high standard for the time period, also, in contrast to almost all

other Indian works composed before and after; the method of the commentary follows a highly systematic order of:

- i. Statement of the rule (sutra)
- ii. Statement of the examples (udaharana)
- iii. Demonstration of the operation (karana) of the rule.

The work is considerably less concise than other Indian works which were often (if not always) written in a poetic form comprising of short statements of rules, and rarely included examples. This poetic form was favoured, not only as it gave the authors an opportunity to demonstrate their skills but also because of the limited supplies of writing equipment available.

The reasons for the composition of the B. Ms. are unknown but it seems possible that the motivation was to "bring out" the developments of mathematics during the time period. Thus it seems very likely it was composed for solely academic, intellectual and interest ends, in short, mathematics for mathematics sake.

By the end of the 2nd century AD mathematics in India had attained a considerable stature, and had become divorced from purely practical and religious requirements, (although it is worth noting that over the next 1000 years the majority of mathematical developments occurred within works on astronomy). The topics of algebra, arithmetic and geometry had developed significantly and it is widely thought that the decimal place value system of notation had been (generally) perfected by 200 AD, the consequence of which was far reaching.

Decimal Numeration and the Place-value System

I have already mentioned on several occasions the development of a decimal place value system of numeration and there is now very little doubt among historians that this invention originated from the Indian subcontinent. That

said, it was considered, until recently, that Arabic scholars were responsible for the system, as C Srinivasiengar writes:

...During the earlier decades of this century (20th) attempts were made to credit this invention wholly or in part to the Arabs. [CS, P 2]

Further attempts have been made to attribute the first use of a place value system to the ancient Babylonian civilisation of Mesopotamia. While it cannot be denied that the Babylonians used a place value system, their's was sexagismal (base 60), and while the concept of place value may have come from Mesopotamia, the Indians were the first to use it with a decimal base (base 10).

All current evidence points towards the Indian system having been influenced by the base 10 Chinese 'counting boards' and the place value system of the Babylonians but combined use of decimal numerals and place value first occurred on the Indian subcontinent. Without doubt the use of a decimal base originates from the most basic human instinct of counting on one's fingers.

The key contribution of the Indians however is not in the development of nine (recognisable) symbols to represent the numbers one to nine, but the invention of the place holder zero.

The great 18th century European mathematician Laplace best described the 'invention' of the decimal place value system as such:

...The idea of expressing all quantities by nine figures whereby is imparted to them both an absolute value and one by position is so simple that this very simplicity is the very reason for our not being sufficiently aware how much admiration it deserves. [CS, P 5]

Beyond not being fully appreciated D Duncan discusses briefly the enduring problem of Eurocentric scholars who

long assumed the symbol for zero was a Greek invention, with no proof at all. The claims were based of pure speculations that zero came from the Greek letter omicron (O), the first letter of the Greek word ouden meaning empty. We know this to be untrue, but it serves as a timely reminder of the struggle for recognition of Indian mathematical developments.

There is wide ranging debate as to when the decimal place value system was developed, but there is significant evidence that an early system was in use by the inhabitants of the Indus valley by 3000 BC. Excavations at both Harappa and Mohenjo Daro have supported this theory. At this time however a 'complete' place value system had not yet been developed and along with symbols for the numbers one through nine, there were also symbols for 10, 20, 100 and so on.

The formation of the numeral forms as we know them now has taken several thousand years, and for quite some time in India there were several different forms. These included Kharosthi and Brahmi numerals, the latter were refined into the Gwalior numerals, which are notably similar to those in use today. Study of the Brahmi numerals has also lent weight to claims that decimal numeration was in use by the Indus civilisation as correlations have been noted between the Indus and Brahmi scripts.

It is uncertain how much longer it took for zero to be invented but there is little doubt that such a symbol was in existence by 500 BC, if not in widespread use.

Evidence can be found in the work of the famous Indian grammarian Panini (5th or 6th century BC) and later the work of Pingala a scholar who wrote a work, Chhandas-Sutra (c. 200 BC).

The first documented evidence of the use of zero for mathematical purposes is not until around 2nd century AD

(in the Bakhshali manuscript). The first recorded 'non-mathematical' use of zero dates even later, around 680 AD, the number 605 was found on a Khmer inscription in Cambodia.

Despite this it seems certain that a symbol was in use prior to that time. B Datta and A Singh discuss the likelihood that the decimal place value system, including zero had been 'perfected' by 100 BC or earlier. Although there is no concrete evidence to support their claims, they are established on the very solid basis that new number systems take 800 to 1000 years to become 'commonly' used, which the Indian system had done by the 9th century AD.

The inventor of the zero symbol is unknown, but what is known is that it was firstly denoted by a dot, then possibly a circle with a dot in the centre, and later by the oval shape we now use.

Prior to its invention, Indian mathematicians had already taken to leaving an empty column on their counting boards and clearly at some point this empty space was filled. The Indians referred to zero as 'sunya' meaning void. Again, although evidence points towards a Mesopotamian origin for a place holder, their 'zero' (two slanted bars) was not used in conjunction with a decimal base.

Having become firmly established in academic circles in India by the 6th century, the decimal place value system spread across the world. Initially to China and Alexandria, then to the Arab empire where it became the system of choice of the scholars in Baghdad by the 8th century.

Arabic scholars during this time improved the system by introducing decimal fractions. The system also spread into Spain, as has been previously discussed southern Spain was under Arabic rule into the 12th century. It took much longer for the system to be accepted in mainland Europe, but eventually by the 16th century it was widely used. That

said, both prejudice and suspicion continued to be widespread, while orthodoxy also played its part in the continued use of Roman numerals. The last significant case of an attempt to abolish the Indian decimal place value system was in Sweden in the early 18th century.

This is clearly a very brief overview of the phenomenal development of the decimal place value system, without which it is accepted 'higher mathematics' would not be possible. It is impossible for me to do justice to its importance in such few words, so I will conclude with a quote from G Halstead who commented:

...The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habituation and a name, a picture, a symbol but helpful power, is the characteristic of the Hindu race from whence it sprang. No single mathematical creation has been more potent for the general on go of intelligence and power. [CS, P 5]

The Classical Period

Introduction

Following the Bakhshali manuscript (assuming it was composed no later than 200 AD) Indian mathematics suffered a slump, described by L Gurjar as:

...A deep and dire degeneration. [LG, P 78]

This was in part due to massive communication problems, but also undoubtedly to the huge political upheaval that took place between the 2nd and 4th centuries AD, prior to the capturing of power of most of India by the Imperial Guptas.

During this period however the practice of writing Siddhantas (astronomical works), which had started around 500 BC, continued. Of the Siddhantas the Pitamaha

Siddhanta is the oldest (c. 500 BC) and the Surya Siddhanta (c. 400 AD, author unknown, influenced Aryabhatta) is the best known. The major contribution of these works was the invention of the sine function. R Gupta comments:

...Trigonometry based on sine (instead of chord (as in Greek works)) and related functions is another important gift of India to the world of mathematics.
[RG2, P 134]

Following the establishment of the 'Gupta dynasty' a period of political stability arose, which doubtlessly contributed to the ascent of a "galaxy" of mathematician-astronomers, led by Aryabhatta. These men were, first and foremost, astronomers, but the mathematical requirements of astronomy (and no doubt further interest) led to them developing many areas of mathematics, as such, I will refer to them just as mathematicians. The vast majority of the works of the 'classical period' were however, in effect, Siddhantas.

Aryabhatta and His Commentators

Aryabhatta, who is occasionally known as Aryabhatta I, or Aryabhatta the elder to distinguish him from a tenth century astronomer of the same name, stands as a pioneer of the revival of Indian mathematics, and the so called 'classical period', or 'Golden era' of Indian mathematics. Arguably the Classical period continued until the 12th century, although in some respects it was over before Aryabhatta's death following a costly, if ultimately successful, war with invading Huns which resulted in the eroding of the Gupta culture (D Duncan P 171). As mentioned, the classical period arose following a 'dark period' of significant political instability 200-400 AD, which caused the widespread stagnation of mathematical development.

We can accurately claim that Aryabhatta was born in 476 AD, as he writes that he was 23 years old when he wrote

his most significant mathematical work the *Aryabhatiya* (or *Arya Bhateeya*) in 499 AD. He was a member of the Kusuma Pura School, but is thought to have been a native of Kerala (in the extreme south of India), although unsurprisingly there is some debate.

Further debate surrounds how important the work of Aryabhatta actually was.

In light of discoveries of both Vedic and Jaina mathematics it has been suggested that his work is of less significance mathematically.

However as Gurjar claims it is more than likely that the majority, if not all prior mathematical work may not have been known to him, which makes his contribution remarkable. Either way, Aryabhatta's work was to have massive influence on those who followed, in a similar way that the work of Luca Pacioli influenced the Italian renaissance mathematicians in the late 15th century.

As previously mentioned the only extant work of Aryabhatta is his key work, the *Aryabhatiya*, a concise astronomical treatise of 118 verses written in a poetic form, of which 33 verses are concerned with mathematical rules.

It is important here to point out that no proofs are contained with his rules, and this is perhaps a primary reason for the neglect by western scholars.

As Indian mathematics is (generally) devoid of proof it is not considered 'true' mathematics in its purest sense. However I believe that adopting this stance is to deny the very origin of remarkable discoveries in mathematics, which may well have been the aim of Eurocentric scholars, as it allowed them to neglect the importance of Indian works in favour of European works.

In the mathematical verses of the *Aryabhatiya* the following topics are covered:

Arithmetic

Method of inversion.

Various arithmetical operators, including the cube and cube root are thought to have originated in Aryabhata's work.

Aryabhata can also reliably be attributed with credit for using the relatively 'new' functions of squaring and square rooting.

Algebra

Formulas for finding the sum of several types of series.

Rules for finding the number of terms of an arithmetical progression.

Rule of three - improvement on Bakshali Manuscript.

Rules for solving examples on interest - which led to the quadratic equation, it is clear that Aryabhata knew the solution of a quadratic equation.

Trigonometry

Tables of sines, not copied from Greek work .

Gupta comments:

...The Aryabhatiya is the first historical work of the dated type, which definitely uses some of these (trigonometric) functions and contains a table of sines.
[RG3, P 72]

Spherical trigonometry (some incorrect).

Geometry

Area of a triangle, similar triangles, volume rules.

It has been suggested that Aryabhata's geometry was borrowed from the Jaina works, but this seems unlikely as it is generally accepted he would not have been familiar with them.

Also of relevance is the use of 'word numerals' and 'alphabet numerals', which are first found in Aryabhatta's work. We can argue that this was not due to the absence of a satisfactory system of numeration but because it was helpful in poetry. C Srinivasiengar quaintly describes it as an:

...Exceedingly queer, if original method of enumeration. [CS, P 43]

However the work of Aryabhatta also affords a proof that:

...The decimal system was well in vogue. [CS, P 43]

Of the mathematics contained within the Aryabhatiya the most remarkable is an approximation for π , which is surprisingly accurate.

The value given is: $\pi = 3.1416$

With little doubt this is the most accurate approximation that had been given up to this point in the history of mathematics.

Aryabhatta found it from the circle with circumference 62832 and diameter 20000. Critics have tried to suggest that this approximation is of Greek origin.

However with confidence it can be argued that the Greeks only used $\pi = 10$ and $\pi = 22/7$ and that no other values can be found in Greek texts.

I note with slight concern for the strength of my last comment that the Egyptian scholar Claudius Ptolemy derived the same value 300 years earlier, although there is no suggestion of a link between these two cases.

Further to deriving this highly accurate value for π , Aryabhatta also appeared to be aware that it was an 'irrational' number and that his value was an approximation, which shows incredible insight.

Thus even accepting that Ptolemy discovered the 4 decimal place value, there is no evidence that he was aware of the concept of irrationality, which is extremely important.

Inexplicably Aryabhatta preferred to use the approximation $= 10 (= 3.1622)$ in practice!

Aryabhatta's work on astronomy was also pioneering, and was far less tinged with a mythological flavour. Among many theories he was the first to suggest that diurnal motion of the 'heavens' is due to rotation of the earth about its axis, which is incredibly insightful (unsurprisingly he was criticised for this.)

In the field of 'pure' mathematics his most significant contribution was his solution to the indeterminate equation:
 $ax - by = c$

R Gupta states:

...Aryabhatta (also) made notable contributions to algebra [RG3, P73]

Although Indian mathematics has become often ignored this was not always the case. The Aryabhatiya was translated into Arabic by Abu'l Hassan al-Ahwazi (before 1000 AD) as Zij al-Arjabhar and it is partly through this translation that Indian computational and mathematical methods were introduced to the Arabs, which will have had a significant effect on the forward progress made by mathematics. The historian A Cajori even goes as far as to suggest that:

...Diophantus, the father of Greek algebra, got the first algebraic knowledge from India. [RG4, P 12]

This theory is supported by evidence that the eminent Greek mathematician Pythagoras visited India, which further 'throws open' the Eurocentric ideal.

EXAMPLE 1:

Solution of $137x + 10 = 60y$, as found in the Aryabhatiya.

The general solution is found as follows:

$$137x + 10 = 60y$$

- 60) 137 (2 (60 divides into 137 twice with remainder 17, etc.)
120

$$17) 60 (3$$

$$51$$

$$9) 17 (1$$

$$9$$

$$8) 9 (1$$

$$8$$

$$1$$

The following column of remainders, known as valli (vertical line) form is constructed:

2

3

1

1

The number of quotients, omitting the first one is 3. Hence we choose a multiplier such that on multiplication by the last residue, 1 (in red above), and subtracting 10 from the product the result is divisible by the penultimate remainder, 8 (in blue above). We have $1 \times 18 - 10 = 1 \times 8$. We then form the following table:

	2	2	2	2	297
	3	3	3	130	130
	1	1	37	37	
	1	19	19		
The multiplier	18	18			
Quotient obtained	1				

This can be explained as such: The number 18, and the number above it in the first column, multiplied and added to the number below it, gives the last but one number in the second column.

Thus, $18 \times 1 + 1 = 19$.

The same process is applied to the second column, giving the third column, that is, $19 \times 1 + 18 = 37$.

Similarly $37 \times 3 + 19 = 130$, $130 \times 2 + 37 = 297$.

Then $x = 130$, $y = 297$ are solutions of the given equation.

Noting that $297 = 23 \pmod{137}$ and $130 = 10 \pmod{60}$, we get $x = 10$ and $y = 23$ as simple solutions.

The general solution is $x = 10 + 60m$, $y = 23 + 137m$.

If we stop with the remainder 8 in the process of division above then we can at once get $x = 10$ and $y = 23$. (Working omitted for sake of brevity).

This method was called Kuttaka, which literally means pulveriser, on account of the process of continued division that is carried out to obtain the solution.

The work of Aryabhata was also extremely influential in India and many commentaries were written on his work (especially his *Aryabhatiya*). Among the most influential commentators were:

Bhaskara I (c 600-680 AD) also a prominent astronomer, his work in that area gave rise to an extremely accurate approximation for the sine function. His commentary of the *Aryabhatiya* is of only the mathematics sections, and he develops several of the ideas contained within. Perhaps his most important contribution was that which he made to the topic of algebra.

Lalla (c 720-790 AD) followed Aryabhata but in fact disagreed with much of his astronomical work. Of note was his use of Aryabhata's improved approximation of π to the

fourth decimal place. Lalla also composed a commentary on Brahmagupta's *Khandakhadyaka*.

Govindasvami (c 800-860 AD) his most important work was a commentary on Bhaskara I's astronomical work *Mahabhaskariya*, he also considered Aryabhata's sine tables and constructed a table which led to improved values.

Shankara Narayana (c 840-900 AD) wrote a commentary on Bhaskara I's work *Laghubhaskariya* (which in turn was based on the work of Aryabhata).

Of note is his work on solving first order indeterminate equations, and also his use of the alternate 'katapayadi' numeration system (as well as Sanskrit place value numerals)

Following Aryabhata's death around 550 AD the work of Brahmagupta resulted in Indian mathematics attaining an even greater level of perfection.

Between these two 'greats' of the classic period lived Yativrsabha, a little known Jain scholar, his work, primarily *Tiloyapannatti*, mainly concerned itself with various concepts of Jaina cosmology, and is worthy of minor note as it contained interesting considerations of infinity.

Brahmagupta, and the Influence on Arabia

Brahmagupta was born in 598 AD, possibly in Ujjain (possibly a native of Sind) and was the most influential and celebrated mathematician of the Ujjain school.

It is important here to note that one must not ignore contributions made by Varahamihira, who was an influential figure at the same Ujjain school during the 6th century. He is thought to have lived from 505 AD till 587 AD and made only fairly small contributions to the field of mathematics, he is described by Ifrah as:

...One of the most famous astrologers in Indian history.
[EFR/JJO'C18, P 1]

However he increased the stature of the Ujjain school while working there, a legacy that was to last for a long period, and although his contributions to mathematics were small they were of some importance.

They included several trigonometric formulas, improvement of Aryabhata's sine tables, and derivation of the Pascal triangle by investigating the problem of computing binomial coefficients.

Returning to Brahmagupta, he not only elaborated the mathematical results of Aryabhata but also made notable contributions to many topics.

L Gurjar describes his results as:

...Unique in the history of world mathematics. [LG, P 91]

His contributions to mathematics are found in two works, the first of which Brahmasphutasiddhanta (BSS) must be considered one of the important mathematical works from this early period, not only of India, but also of the world. Not only was its mathematical content of an exceptional quality, but the work also had a significant influence on the burgeoning scientific awakening in the Arab empire.

I believe that the Indian influence on Arabic work is often ignored or played down and consider this to be unfortunate (at the least). This issue is definitely worthy of discussion as it is noticeable that much is made of the Greek influence on Arabic works but far less of the Indian influence, which in retrospect was quite significant.

His second work was written much later in his life in 665 AD and was titled Khandakhayaka. Although the BSS contains 25 chapters it is generally considered that the first ten chapters make up the first work, and that at a later date Brahmagupta made revisions and additions.

In the BSS among the major developments are those in the areas of:

Arithmetic

Brahmagupta possessed a greater understanding of the number system (and place value system) than anyone to that point. Many rules are given and an advanced technique for multiplication exhibited.

Operations with zero, Brahmagupta was the first to attempt to divide by zero, and while his attempts; showing $n/0 =$, were not ultimately successful they demonstrate an advanced understanding of an extremely abstract concept.

Operations with negative numbers.

Theory of Arithmetic progressions.

Algorithm for calculating square roots that is equivalent to the Newton-Raphson iterative formula, but clearly pre-dates it by many centuries.

Geometry

Brahmagupta stands in high esteem for his contributions to this topic. A rule that he gives for finding the values of the diagonals of cyclic quadrilaterals is generally known as "Ptolemy's theorem".

Ptolemy 'pre-dated' Brahmagupta by 500 years, so it is wholly reasonable to attribute the 'discovery' of these rules to him. However, Brahmagupta's independent discovery should still be considered a remarkable achievement.

Furthermore, some of his work (regarding right angled triangles, which was later developed by Mahavira, Bhaskara II, et al) is often attributed to Fibonacci (13th c.) and Vieta (16th c.), highlighting the constant European bias.

As L Gurjar quotes:

...(Brahmagupta) derived certain results, which were troubling the brains of Western mathematicians as late as the 17th century. [LG, P 91]

Algebra

Solutions to $N_x^2 + 1 = y^2$, Pell's equation, his most outstanding contribution to mathematics.

He also made many other contributions to solving a variety of algebraic equations, including $ax + c = by$ (which is the focus of a paper by P Majumdar).

Brahmagupta may have been one of the first mathematicians to recognise that the quadratic equation has two solutions.

In his other work, the content is far more 'pure' astronomy, but an interpolation formula used to calculate values of sines bears great similarity to the Newton-Stirling interpolation formula, which is clearly of great historical and mathematical interest.

Without a doubt, Brahmagupta made remarkable contributions to mathematics (and astronomy) and his work continued to be influential for many centuries. In 860 AD an extensive and important commentary on the BSS was written by Prthudakasvami (or Prithudaka Swami). His work was extremely elaborate and unlike many Indian works did not 'suffer' brevity of expression.

The spread of Buddhism (around 500 AD) into China resulted in a period of cultural and scientific exchanges lasting several centuries. Chinese scholars are known to have translated the work of Brahmagupta; this highlights not only the quality of the work but the influence it had on the world outside India. R Gupta mentions four 'Brahminical' translations in a paper. During this time the decimal system and notation was adopted by Chinese scholars and as R Gupta states:

...Indian mathematical astronomy exerted a great influence in China during the (glorious) Thang Period (618-907). [RG4, P 11]

The lasting legacy of the BSS however was its translation by Arab scholars and its contribution to the 'forward progress' of mathematics. These translations, along with translated work of Aryabhatta and (possibly) the Surya Siddhanta were responsible for alerting the Arabs, and the West to Indian mathematics (and astronomy), as G Joseph states:

...This was to have momentous consequences for the development of the two subjects. [GJ, P 267]

Of particular interest is the well told story of the Indian scholar who traveled to Baghdad, at the behest of Caliph al-Mansur (early ruler of the Arab Empire). R Gupta reports the story as such:

...In the year 156 (772/773 AD) there came to Caliph al-Mansur a man (an Ujjain scholar by the name of Kanka) from India, an expert in hisab (computation) bringing with him a work called Sindhind (i.e. Siddhanta) concerning the motions of the planets. [RG, P 12]

A translation of this work, thought to be Brahmagupta's BSS, was subsequently carried out by al-Fazari (and an Indian scholar) and had a far-reaching influence on subsequent Arabic works. The famous Arabic scholar al-Khwarizmi (credited with 'inventing' algebra) is known to have made use of the translation, called Zij al-Sindhind.

Al-Khwarizmi (c. 780-850 AD) is known to have written two subsequent works, one based on Indian astronomy (Zij) and the other on arithmetic (possibly Kitab al-Adad al-Hindi). Later Latin translations of this second work (Algorithmi De Numero Indorum), composed in Spain around the 11th century, are thought to have played a crucial role in introducing the Indian place-value system numerals and the corresponding computational methods into (wider) Europe. Both Indian astronomy and arithmetic had a huge impact in Spain.

This discussion helps to highlight the influence that Indian mathematics had on Arabic mathematics, and ultimately, through Latin translations, on European mathematics, an influence that is considerably neglected. It must be argued that sufficient credit has not been given.

There seems to have been relatively little further 'interaction' between Indian and Arab scholars, and thus Indian works had limited, if any, further influence on mathematical developments in other countries. However Indian mathematics continued to flourish independently throughout the subcontinent for another 400 years, and some of the most outstanding contributions to the history of world mathematics were in fact made during this time period.

Mathematics Over the Next 400 Years (700AD-1100AD)

Mahavira (or Mahaviracharya), a Jain by religion, is the most celebrated Indian mathematician of the 9th century. His major work *Ganitasar Sangraha* was written around 850 AD and is considered 'brilliant'. It was widely known in the South of India and written in Sanskrit due to his Jaina 'faith'. In the 11th century its influence was still being felt when it was translated into Telegu (a regional language of the south). Mahavira was aware of the works of Jaina mathematicians and also the works of Aryabhatta (and commentators) and Brahmagupta, and refined and improved much of their work. What makes Mahavira unique is that he was not an astronomer, his work was confined solely to mathematics and he stands almost entirely alone in the history of Indian mathematics (at least up to the 14th century) in this respect. He was a member of the mathematical school at Mysore in the south of India and his major contributions to mathematics include:

Arithmetic: GSS was the first text on arithmetic in the present form. He made the classification of arithmetical operators simpler. Detailed operations with fractions (and

unit fractions), but no section on decimals (which were not an Indian invention).

Geometric progressions—he gave almost all required formulae.

Permutations and Combinations: Extension and systemisation of Jain works. First to give general formula.

Geometry: Repeated Brahmagupta's construction for cyclic quadrilaterals.

Definitions for most geometric shapes.

Algebra: Work on quadratic, indeterminate and simultaneous equations. Mahavira demonstrated definite understanding of the concept of a quadratic equation having two roots.

Ellipse: Only Indian mathematician to refer to the ellipse, indeed Indian mathematicians did not study conic sections or anything along these lines. Gave incorrect identity for area of ellipse. His formula for the perimeter of an ellipse is worth noting.

Mahavira's work, GSS, could be criticised for being nothing more than an extensive commentary on Jaina works, and the work of Aryabhatta, Brahmagupta (and Bhaskara I). C Srinivasiengar describes his work as containing:

...(No) profoundly fundamental discoveries. [CS, P 70]

To some extent this may be true, but it is also unfair. Mahavira made many subtle contributions and elaborated and revised much of the work of previous mathematicians. Furthermore GSS contained many examples to illustrate his rules, unlike many Indian mathematical works.

The influence of his work (within India) must also not be ignored. The very fact it was still in use more than 250 years after it was written is testimony to its importance as a mathematical work.

EXAMPLE 1:

General formula for combinations, as given by Mahavira.

$${}_nC_r = \{n(n-1)(n-2)\times\times(n-r+1)\}/1.2.3. \dots r$$

Following Mahavira the most notable mathematician was Prthudakasvami (c. 830-890 AD) a prominent Indian algebraist, who is described by E Robertson and J O'Connor as being:

...Best known for his work on solving equations. [EFR/JJO'C25, P 1]

He also wrote a commentary on Brahmagupta's *Brahma Sputa Siddhanta*.

In the early 10th century a mathematician by the name of Sridhara (c. 870-930 AD) may have lived, however there is much debate surrounding his birth and some authors place him as having lived in the 8th century (750 AD). However beyond debate is the fact that he wrote *Patiganita Sara* a work on arithmetic and mensuration. It contained exactly 300 verses and is hence also known by the name *Trisatika*. It includes contributions to the following topics:

Rules on extracting square and cube roots, fractions and eight rules for operations involving zero (not division).

Theory of cyclic quadrilaterals with rational sides.

A section concerning rational solutions of various equations of the Pell's type.

Methods for summation of different arithmetic and geometric series. These methods became standard references in later works.

It is thought that Sridhara also composed a text on algebra, which is now lost, and several other works have been attributed to Sridhara, but there is no certainty if they were indeed written by him. The legacy of Sridhara's

work was that it had some influence on the work of Bhaskaracharya II, regarded by many as the greatest Indian mathematician.

Prior to Bhaskara II it is worth noting the contributions of Aryabhata II (c 920-1000 AD) a mathematical-astronomer who notably made important contributions to algebra. In his work *Mahasiddhanta* he gives twenty verses of detailed rules for solving $by = ax + c$ (and variations of this equation). Also of note, Vijayanandi (c 940-1010 AD) who made several contributions to trigonometry in the course of his astronomical works, and Sripati.

Sripati (c. 1019-1066 AD, the birth year of 1019 is known to be correct) was a follower of the teachings of Lalla and in fact the most important Indian mathematician of the 11th century. He was the author of several astronomical works, including *Siddhantasekhara*, which contained two chapters devoted to mathematics. His major contributions were in the fields of arithmetic and algebra. His algebra is of particular note; his work includes rules for solving the quadratic equation and simultaneous indeterminate equations.

Further he impressively gave the identity:

$$\sqrt{(x + \sqrt{y})} = \sqrt{\left[\frac{x + \sqrt{(x^2 - y)}}{2}\right]} + \sqrt{\left[\frac{x - \sqrt{(x^2 - y)}}{2}\right]}$$

Bhaskaracharya II

Bhaskaracharya, or Bhaskara II, is regarded almost without question as the greatest Hindu mathematician of all time and his contribution to not just Indian, but world mathematics is undeniable. As L Gurjar states:

...Because of his work India gave a definite 'quota' to the forward world march of the science. [LG, P 104]

Born in 1114 AD (in Vijayapura, he belonged to Bijjada Bida) he became head of the Ujjain school of mathematical

astronomy (Varahamihira and Brahmagupta had helped to found this school or at least 'build it up').

There is some confusion amongst the texts I have referred to as to the works that he wrote. C Srinivasiengar claims he wrote Siddhanta Siromani in 1150 AD, which contained four sections:

- (1) Lilavati (arithmetic)
- (2) Bijaganita (algebra)
- (3) Goladhyaya (sphere/celestial globe)
- (4) Grahaganita (mathematics of the planets)

E Robertson and J O'Connor claim that he wrote 6 works, 1), 2) and SS (which contained two sections) and three further astronomical works, including two commentaries on the SS.

G Joseph claims his mathematically significant works were 1), 2), and SS (which indeed he wrote in 1150 and is a highly influential astronomical work). S Sinha however agrees with C Srinivasiengar that Lilavati was a section (chapter) of the SS, and thus I will agree with the respected Indian historians.

Lilavati (or Leelavati, there is a charming if unlikely story regarding the origin of the name of this work) is divided into 13 chapters (possibly by later scribes) and covers many branches of mathematics, arithmetic, algebra, geometry, and a little trigonometry and mensuration.

More specifically the contents include:

- Definitions.
- Properties of zero (including division).
- Further extensive numerical work, including use of negative numbers and surds.
- Estimation of.

- Arithmetical terms, methods of multiplication, squaring, inverse rule of three, plus rules of 5, 7 and 9.
- Problems involving interest.
- Arithmetical and geometrical progressions.
- Plane geometry.
- Solid geometry.
- Combinations.
- Indeterminate equations (Kuttaka), integer solutions (first and second order) His contributions to this topic are among his most important, the rules he gives are (in effect) the same as those given by the renaissance European mathematicians (17th Century) yet his work was of 12th Century. Method of solving was an improvement of the methods found in the work of Aryabhatta and subsequent mathematicians.
- Shadow of the gnomon.

The Lilivati is written in poetic form with a prose commentary and Bhaskara acknowledges that he has condensed the works of Brahmagupta, Sridhara (and Padmanabha).

However his work is outstanding for its systemisation, improved methods and the new topics that he has introduced. Furthermore the Lilavati contained excellent recreative problems and it is thought that Bhaskara's intention may have been that a student of 'Lilavati' should concern himself with the mechanical application of the method. A student of 'Bijaganita' should however concern himself with the theory underlying the method.

His work Bijaganita is effectively a treatise on algebra and contains the following topics:

- Positive and negative numbers.

- Zero.
- The 'unknown'.
- Surds.
- Kuttaka.
- Simple equations (indeterminate of second, third and fourth degree).
- Simple equations with more than one unknown.
- Indeterminate quadratic equations (of the type $ax^2 + b = y^2$).
- Quadratic equations.
- Quadratic equations with more than one unknown.
- Operations with products of several unknowns.

Bhaskara derived a cyclic, 'Cakraval' method for solving equations of the form $ax^2 + bx + c = y$, which is usually attributed to William Brouncker who 'rediscovered' it around 1657. Bhaskara's method for finding the solutions of the problem $Nx^2 + 1 = y^2$ (so called "Pell's equation") is of considerable interest and importance.

His work the Siddhanta Siromani is an astronomical treatise and contains many theories not found in earlier works. There is not a large mathematical content, but of particular interest are several results in trigonometry and calculus that are found in the work. These include results of differential and integral calculus.

Bhaskara is though to be the first to show that:

$$\delta \sin x = \cos x \quad \delta x$$

Evidence suggests Bhaskara was fully acquainted with the principle of differential calculus, and that his researches were in no way inferior to Newton's, besides the fact that it seems he did not understand the utility of his researches, and thus historians of mathematics generally neglect his outstanding achievement, which is extremely regrettable.

Bhaskara also goes deeper into the 'differential calculus' and suggests the differential coefficient vanishes at an extremum value of the function, indicating knowledge of the concept of 'infinitesimals'.

He also gives the (now) well known results for $\sin(a + b)$ and $\sin(a - b)$. There is also evidence of an early form of Rolle's theorem;

if $f(a) = f(b) = 0$ then $f'(x) = 0$

for some x with $a < x < b$,

in Bhaskara's work.

There have been several unscrupulous attempts to argue that there are traces of Diophantine influence in Bhaskara's work, but this once again seems like an attempt by European scholars to claim European influence on (all) the great works of mathematics. These claims should be ignored. Particularly in the field of algebra, Diophantus only looked at specific cases and did not achieve the general methods of the Indians.

Pell's Equation

Although there were great advancements in many areas of algebra, and the solution of many different forms of equations, from simple forms such as $ax + c = by$, to forms as complex as $ax^2 + bxy + cy^2 = z^2$, I have chosen to look in slightly more detail at solution(s) to the so called Pell's equation, $Nx^2 + 1 = y^2$. The equation was so called due to a mistake on the part of Euler, who attributed the solution of the equation to John Pell, a 17th century English scholar, who actually only referred to the equation in a text he wrote on algebra.

The equation was nearly solved by Brahmagupta (628 AD) and the solution was improved by Bhaskara II (1150 AD), leading some historians, including C Srinivasiengar, to suggest:

...It is therefore fitting that this equation be called the Brahmagupta-Bhaskara equation. [CS, P 110]

The complete theory underlying the solution was expounded by Lagrange in 1767, and rests on the theory of continued fractions. It must be briefly noted how remarkable the achievements of Indian scholars were, given the time period in which equations of the Pell's type were studied. The Indian method involves an element of trail-process but contains no mention of continued fractions.

Further to solving equations of the Pell's type to obtain solutions for the unknowns, Brahmagupta extended his method of solution to find square roots. This contribution is of huge interest as it is essentially the same method rediscovered and used by Newton and Raphson around 1690, which is known as the Newton-Raphson iterative method.

Contained within this brief discussion is a small computer code for the Maple mathematics package, which uses the Brahmagupta type solution of Pell's equation to derive extremely accurate square roots.

The Pell's type of equation was known in India as Varga Prakriti, or "equation of the multiplied square", where prakriti means coefficient and refers to the coefficient N (where N is a positive integer). As previously mentioned, Bhaskara developed a Chakravala or cyclic method of solution.

Regrettably, due to constraints of space, I will have to forego discussion of the general solutions derived by Brahmagupta and Bhaskara, and will include only an example to illustrate Bhaskara's improved 'cyclic' method.

The following example is of great historical interest. It is found in the Bijaganita of Bhaskara and is also of the form of a problem Fermat set as a problem to fellow mathematician Frenicle in 1657. The smallest solution for x and y runs into 4 and 5 digits respectively. The chakravala method is

remarkable, as it requires just a few 'easy' steps, while Lagrange's solution required complex use continued fractions.

EXAMPLE 1:

Solution of $67x^2 + 1 = y^2$.

$$67x^2 + 1 = y^2.$$

Firstly the auxiliary equation $67 \times 12 - 3 = 82$ is taken.

Then using Bhaskara's lemma, where $Na^2 + k = b^2$, where a, b, k are the integers (1, -3 and 8) in the auxiliary equation above, (k being positive or negative) then:

$$N((am + b)/k)^2 + ((m^2 - N)/k) = ((bm + Na)/k)^2$$

Thus:

$$67((1 \times m + 8)/-3)^2 + ((m^2 - 67)/-3) = ((8m + 67)/-3)^2 \dots(1)$$

Then, by the method of Kuttaka the solution of

$$(m + 8) \cdot -3 = \text{an integer, is } m = -3t + 1.$$

Putting $t = -2$, we get $m = 7$, which makes $[m^2 - 67]$ least.

On substituting this value, the equation (1) reduces to:

$$67 \times 52 + 6 = 412$$

Again, by the lemma:

$$67((5n + 41)/6)^2 + ((n^2 - 67)/6) = ((41n + 67)/6)^2 \dots(2)$$

Then the solution of $(5n + 41)/6 = \text{a whole number}$, is $n = 6t + 5$. $[n^2 - 67]$ will be least for the value $t = 0$, that is, when $n = 5$. The equation (1) then becomes:

$$67 \times 112 - 7 = 902$$

Now we form:

$$67((11p + 90)/-7)^2 + ((p^2 - 67)/-7) = ((90p + 67)/-7)^2 \dots(3)$$

The solution of $(11p + 90)/-7 = \text{an integral number}$, is $p = -7t + 2$. Taking $t = -1$, we have

$p = 9$; and this value makes $[p^2 - 67]$ least. Substituting that into (3) we get:

$$67 \times 272 - 2 = 2212$$

By the Principle of Composition of Equals, we get from the above equation:

$$67(2.27.221)^2 + 4 = (2212 + 67 \times 272)^2$$

$$\text{Or } 67(11934)^2 + 4 = (97684)^2$$

Dividing out by 4 we have:

$$67(5967)^2 + 1 = (48842)^2$$

Hence $x = 5967$, $y = 48842$ is a solution of the equation $67x^2 + 1 = y^2$. (See C Srinivasiengar Ps 110-133 for further details on Indian solutions to Pell's equation.)

I will conclude my brief discussion of Pell's equation by illustrating Brahmagupta's method for calculating square roots with the following example. The method was used by Brahmagupta to find the square root of the integer N , when $Nx^2 + 1 = y^2$, and solutions for x and y are known.

EXAMPLE 2:

Finding the square root of N , from $Nx^2 + 1 = y^2$

If $N = 5$, then $y^2 = 1 + 5x^2$. It can be observed that $5 = (y^2 - 1)/x^2$ and that $(y^2 - 1)/x^2 \gg y^2/x^2$.

This is the key to finding the square root of 5, as $5 = y/x$. With ease we can identify $y = 9$ and $x = 4$ as solutions to this equation, and we see $\sqrt{5} \gg 9/4 = 2.25$, ($\sqrt{5} = 2.236067978\dots$).

Clearly the larger y and x are, the better the approximation is. This can be shown using the following Maple programme:

```
> n:=5:
```

```
> f:=(x,y) => (2*x*y, y*y + n*x*x):
```

```
> m:=0:
```

```
> x:=4
```



```

> y:=9
> while m 5 do
> m:= m + 1;
> print (x,y,evalf(y/x,20), evalf(y/x-sqrt(n),50));
> a:=f (x,y);
> x:=a [1];
> y:=a [2];
> end do:

```

The output gives the following:

4, 9 (solutions of x and y)

2.25000000000000000000 (y/x to 20 decimal places)

0.0139320225002103035908263312687237645593816403885

(error between y/x and $\sqrt{5}$)

By the 5th step the following result is given:

25840354427429161536, 57780789062419261441

(5th pair of solutions for x and y)

2.2360679774997896964 (y/x to 20 d.p.s)

$0.3348791201 \times 10^{-39}$ (error)

This result is extremely accurate, and the method must be considered brilliant given it was derived by Brahmagupta in 628 AD.

The End of the Classic Period

The work of Bhaskara was considered the highest point Indian mathematics attained, and it was long considered that Indian mathematics ceased after that point. Extreme political turmoil through much of the sub-continent shattered the atmosphere of discovery and learning and led to the stagnation of mathematical developments as scholars contented themselves with duplicating earlier works.

Recent discoveries however have found that, despite political turmoil, mathematics continued to a high degree in the south of India up to the 16th century. The South of India avoided the worst of the political upheavals of the subcontinent, and the Kerala School of mathematics flourished for some time, producing some truly remarkable results. These results, the most notable of which are in the field of infinite series expansions of trigonometric functions, are generally inaccurately attributed to great European mathematicians of the 18th century including Newton, Leibniz and Gregory. However, slowly, this rigid position is shifting somewhat.

Before going on to discuss the Kerala contribution to mathematics it is worth noting that by the time of Bhaskara II's death Indian mathematics of the 5th and 6th centuries had exerted a significant influence on mathematics across the world. By the 11th century a number of important Arabic works had been written, based on translations of a number of Indian astronomical works. As the Arab Empire stretched as far as southern Spain, much of this work based on Indian science made its way into southern Europe and was subsequently translated into Latin.

Sadly there is very little recognition of these facts, and even though the Arabic (and hence some Indian) works were prevalent in Spain, they did not transmit any further into Europe, which was still to fully 'awaken and probably 'resisted' the works, and many were subsequently lost. However ultimately a few Latin translations of Indo-Arabic works did flow into wider Europe, causing a step towards the renaissance.

This brief return to this discussion is primarily to serve as a reminder that Indian mathematics has had a far greater influence on the forward progress of mathematics (in conjunction with enlightened Arab scholars, and ultimately a handful of pioneering European scholars) than is generally

mentioned. A prime example is the work of Fibonacci, which shows appreciation of Indo-Arabic work as early as the 12th century. In short, around the time of Bhaskara's death (12th c.) Indian mathematics (of the 5th-7th centuries) was still exerting a significant influence throughout the world.

As mentioned, it was long considered that following the 'high point' of the work of Bhaskara that Indian mathematics fell into a steep decline. To some extent this is true, only shortly after Bhaskara's death India was engulfed in war (Mongol invasions) and political turmoil. Consequently the atmosphere of security and tranquillity was lost, which was doubtlessly a major contributory factor to the barrenness of scientific activity and achievement. As C Srinivasiengar quotes:

...India suddenly fell into a state of "torpor" and never recovered from this torpor until the advent of mathematicians trained according to Western methods. [CS, P 142]

There were occasional small developments, and attempts to revive learning, but nothing of the magnitude of the previous millennium.

Worthy of a brief mention are both Kamalakara (c 1616-1700) and Jagannatha Samrat (c. 1690-1750). Both combined traditional ideas of Indian astronomy with Arabic (and some Greek) concepts, Kamalakara gave trigonometric results of interest and Samrat made several Sanskrit translations of Arabic 'versions' of Greek works, including notably Euclid's Elements. However, as C Srinivasiengar comments:

...Jagannatha's work was not mere translation. [CS, P 143]

Indeed his work contained new proofs not given by Euclid.

Under the patronage of monarch Sawai Jayasinha Raja (and his predecessor) Samrat was attempting, along with a group of scholars, to 'reinvigorate' science and learning in

India. It must be admitted that these efforts do not appear to have been wholly successful, although the efforts were in the 'greatest of faith' and should be 'applauded'. To some extent the work of these two scholars only serves to further highlight the lack of originality and indolent attitude that was present by this stage in the north of India, although their efforts should not be completely ignored.

It must be considered most unfortunate that a country, which on reflection was unarguably a world leader in the field of mathematics for several thousands of years, ceased to contribute in any significant way. A theory has been suggested that, had there been definite 'links' between each 'major' period we have discussed, then India would have led the world, unequivocally, in the field of mathematics and may have continued to for much longer. However discoveries that have been made in the last 150 years have significantly altered the chronology of Indian mathematics and the way in which we should view Indian contributions.

Keralese Mathematics I

The south western tip of India escaped the majority of the political upheaval, which engulfed the rest of the country, allowing a generally peaceful existence to continue. Thus the pursuit of scientific development was able to continue 'uninterrupted'. It has only recently come to light that mathematics (and astronomy) continued to flourish in this area for several hundred years. Kerala mathematics was strongly influenced by astronomy, but this led to the derivation of mathematical results of huge importance. As a result of the recency of these discoveries it is quite probable that there are still further discoveries of 'Kerala mathematics' to be made, and a full analysis has yet to be carried out. However several findings have already been made that show several major concepts of renaissance European mathematics were first developed in India.

Indeed G Joseph quotes:

...In Kerala, the period between the 14th and 16th centuries marked a high point in the indigenous development of (astronomy) and mathematics. [GJ, P 287]

The works that have so far been analysed are of such a high level that it is though there may be missing links between the "classical period" and the medieval period of Kerala. There is also interest in the claims that European scholars may have had first hand knowledge of some Kerala mathematics, as the area was a focal point for trading with many parts of the world, including Europe. There is also some evidence of a transfer of technology between Europe and Kerala. I will discuss this issue in a little more depth later.

At this point in time only some interest is being paid to the recent discoveries that have been made, highlighting that in many historical 'circles' Indian developments are still not considered important. A further point of interest is that up to the 10th century there was virtually no mathematical activity of note in the south of India. There are four areas in the south of the country - including Kerala in the south west corner, and with the exception of Mahavira (resident of Karnataka area) there was no mathematical output of significance until the 11th-12th century early Kerala literature. There are however impressions that Aryabhata I was a Keralite, and indeed, if this were true, then in the words of K Rajagopalan:

...Kerala would find a prominent place in the mathematical map of "our" country. [KR1, P 81]

Bhaskara I is also thought to have possibly been a Keralite.

Of the leading mathematicians of Kerala there is quite possibly more to be discovered but currently there are several whose work is of significant interest.

Mathematicians of Kerala

Narayana Pandit (c. 1340-1400), the earliest of the notable Keralese mathematicians, is known to have definitely written two works, an arithmetical treatise called *Ganita Kaumudi* and an algebraic treatise called *Bijganita Vatamsa*. He was strongly influenced by the work of Bhaskara II, which proves work from the classic period was known to Keralese mathematicians and was thus influential in the continued progress of the subject. Due to this influence Narayana is also thought to be the author of an elaborate commentary of Bhaskara II's *Lilavati*, titled *Karmapradipika* (or *Karma-Paddhati*). It has been suggested that this work was written in conjunction with another scholar, Shankara Variyar, while others attribute the work to Madhava (see later).

Although the *Karmapradipika* contains very little original work, seven different methods for squaring numbers are found within it, a contribution that is wholly original to the author. Narayana's other major works contain a variety of mathematical developments, including a rule to calculate approximate values of square roots, using the second order indeterminate equation $Nx^2 + 1 = y^2$ (Pell's equation). Mathematical operations with zero, several geometrical rules and discussion of magic squares and similar figures are other contributions of note. Evidence also exists that Narayana made minor contributions to the ideas of differential calculus found in Bhaskara II's work.

R Gupta has also brought to light Narayana's contributions to the topic of cyclic quadrilaterals. Subsequent developments of this topic, found in the works of Shankara Variyar and Ganesa interestingly show the influence of work of Brahmagupta.

Parameshvara (c. 1370-1460) is known to have been a pupil of Narayana Pandit, and also Madhava of Sangamagramma, who I will discuss later and is thought to have been a significant influence. He wrote commentaries

on the work of Bhaskara I, Aryabhata I and Bhaskara II, and his contributions to mathematics include an outstanding version of the mean value theorem. Furthermore Parameshvara gave a mean value type formula for inverse interpolation of sine, and is thought to have been the first mathematician to give the radius of circle with inscribed cyclic quadrilateral, an expression that is normally attributed to Lhuillier (1782).

In turn, Nilakantha Somayaji (1444-1544) was a disciple of Parameshvara and was educated by his son Damodra. In his most notable work *Tantra Samgraha* (which 'spawned' a later anonymous commentary *Tantrasangraha-vyakhya* and a further commentary by the name *Yuktidipaika*, written in 1501) he elaborates and extends the contributions of Madhava.

Sadly none of his mathematical works are extant, however it can be determined that he was a mathematician of some note. Nilakantha was also the author of *Aryabhatiya-bhasa* a commentary of the *Aryabhatiya*. Of great significance is the presence of mathematical proof (inductive) in Nilakantha's work.

Furthermore, his demonstration of particular cases of the series

$$\tan^{-1} t = t - t^3/3 + t^5/5 - \dots,$$

when $t = 1$ and $t = 1/\sqrt{3}$, and remarkably good rational approximations of (using another Madhava series) are of great interest. Various results regarding infinite geometrically progressing convergent series are also attributed to Nilakantha.

Citabhanu (1475-1550) has yet to find a place in books on Indian mathematics. His work on the solution of equations is quoted in a work called *Kriya-krama-kari*, by the scholar Shankara Variar, who is also relatively little known (although R Gupta mentions a further text, written by him).

Jyesthadeva (c. 1500-1575) was a member of the Kerala School, which was founded on the work of Madhava, Nilakantha, Parameshvara and others. His key work was the *Yukti-bhasa* (written in Malayalam, a regional language of Kerala). Similarly to the work of Nilakantha it is almost unique in the history of Indian mathematics, in that it contains both proofs of theorems and derivations of rules. He also studied various topics found in many previous Indian works, including integer solutions of systems of first degree equations solved using *kuttaka*.

Madhava of Sangamagramma

Although born in Cochin on the Keralese coast before the previous four scholars I have chosen to save my discussion of Madhava of Sangamagramma (c. 1340 - 1425) till last, as I consider him to be the greatest mathematician-astronomer of medieval India. Sadly all of his mathematical works are currently lost, although it is possible extant work may yet be 'unearthed'. It is vaguely possible that he may have written *Karana Paddhati* a work written sometime between 1375 and 1475, but this is only speculative. All we know of Madhava comes from works of later scholars, primarily Nilakantha and Jyesthadeva. G Joseph also mentions surviving astronomical texts, but there is no mention of them in any other text I have consulted.

His most significant contribution was in moving on from the finite procedures of ancient mathematics to 'treat their limit passage to infinity', which is considered to be the essence of modern classical analysis. Although there is not complete certainty it is thought Madhava was responsible for the discovery of all of the following results:

- (1) $\theta = \tan \theta - (\tan^3 \theta)/3 + (\tan^5 \theta)/5 - \dots$, equivalent to Gregory series.
- (2) $r\theta = \{r(r\sin \theta)/1(r\cos \theta)\} - \{r(r\sin \theta)^3/3(r\cos \theta)^3\} + \{r(r\sin \theta)^5/5(r\cos \theta)^5\} - \dots$

(3) $\sin \theta = \theta - \theta^3/3! + \theta^5/5! - \dots$, Madhava-Newton power series.

(4) $\cos \theta = 1 - \theta^2/2! + \theta^4/4! - \dots$, Madhava-Newton power series.

Remembering that Indian $\sin \theta = r \sin \theta$, and Indian $\cos \theta = r \cos \theta$. Both the above results are occasionally attributed to Maclaurin.

(5) $\pi/4 \cong 1 - 1/3 + 1/5 - \dots \pm 1/n \pm (-f_i(n+1))$, $i = 1, 2, 3$, and where $f_1 = n/2$, $f_2 = (n/2)/(n^2 + 1)$ and $f_3 = ((n/2)^2 + 1)/((n/2)(n^2 + 4 + 1))^2$ (a power series for π , attributed to Leibniz)

(6) $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots \pm 1/n \pm \{-f(n+1)\}$, Euler's series.

A particular case of the above series when $t = 1/\sqrt{3}$ gives the expression:

(7) $p = \sqrt{12} (1 - \{1/(3 \times 3)\} + \{1/(5 \times 3^2)\} - \{1/(7 \times 3^3)\} + \dots)$

In generalisation of the expressions for f_2 and f_3 as continued fractions, the scholar D Whiteside has shown that the correcting function $f(n)$ which makes 'Euler's' series (of course it is not in fact Euler's series) exact can be represented as an infinite continued fraction. There was no European parallel of this until W Brouncker's celebrated reworking in 1645 of J Wallis's related continued product.

A further expression involving :

(8) $\pi d \cong 2d + 4d/(2^2 - 1) - 4d/(4^2 - 1) + \dots \pm 4d/(n^2 + 1)$ etc., this resulted in improved approximations of π , a further term was added to the above expression, allowing Madhava to calculate π to 13 decimal places. The value $p = 3.14159265359$ is unique to Kerala and is not found in any other mathematical literature. A value correct to 17 decimal places (3.14155265358979324) is found in the work

Sadratnamala. R Gupta attributes calculation of this value to Madhava, (so perhaps he wrote this work, although this is pure conjecture).

Of great interest is the following result:

- (9) $\tan^{-1}x = x - x^3/3 + x^5/5 - \dots$, Madhava-Gregory series, power series for inverse tangent, still frequently attributed to Gregory and Leibniz.

It is also expressed in the following way:

- (10) $\text{rarctan}(y/x) = ry/x - ry^3/3x^3 + ry^5/5x^5 - \dots$, where $y/x \leq 1$

The following results are also attributed to Madhava of Sangamagramma:

$$(11) \sin(x + h) \cong \sin x + (h/r)\cos x - (h^2/2r^2)\sin x$$

$$(12) \cos(x + h) \cong \cos x - (h/r)\sin x - (h^2/2r^2)\cos x$$

Both the approximations for sine and cosine functions to the second order of small quantities, (see over page) are special cases of Taylor series, (which are attributed to B Taylor).

Finally, of significant interest is a further 'Taylor' series approximation of sine:

$$(13) \sin(x + h) \cong \sin x + (h/r)\cos x - (h^2/2r^2)\sin x + (h^3/6r^3)\cos x.$$

Third order series approximation of the sine function usually attributed to Gregory.

With regards to this development R Gupta comments:

...It is interesting that a four-term approximation formula for the sine function so close to the Taylor series approximation was known in India more than two centuries before the Taylor series expansion was discovered by Gregory about 1668. [RG5, P 289]

Although these results all appear in later works, including the Tantrasangraha of Nilakantha and the Yukti-bhasa of

Jyesthadeva it is generally accepted that all the above results originated from the work of Madhava. Several of the results are expressly attributed to him, for example Nilakantha quotes an alternate version of the sine series expansion as the work of Madhava. Further to these incredible contributions to mathematics, Madhava also extended some results found in earlier works, including those of Bhaskaracarya.

The work of Madhava is truly remarkable and hopefully in time full credit will be rewarded to his work, as C Rajagopal and M Rangachari note:

...Even if he be credited with only the discoveries of the series (sine and cosine expansions, see above, 3) and 4)) at so unexpectedly early a date, assuredly merits a permanent place among the great mathematicians of the world. [CR /MR1, P 101]

Similarly G Joseph states:

...We may consider Madhava to have been the founder of mathematical analysis. Some of his discoveries in this field show him to have possessed extraordinary intuition. [GJ, P 293]

With regards to Keralese contributions as a whole, M Baron writes (in D Almeida, J John and A Zadorozhnyy):

...Some of the results achieved in connection with numerical integration by means of infinite series anticipate developments in Western Europe by several centuries. [DA/JJ/AZ1, P 79]

There remains a final Kerala work worthy of a brief mention, Sadrhana-Mala an astronomical treatise written by Shankara Varman serves as a summary of most of the results of the Kerala School. What is of most interest is that it was composed in the early 19th century and the author stands as the last notable name in Keralese mathematics.

In recent histories of mathematics there is acknowledgement that some of Madhava's remarkable results were indeed first discovered in India. This is clearly a positive step in redressing the imbalance but it seems unlikely that full 'credit' will be given for some time, as that will possibly require the re-naming of various series, which seems unlikely to happen!

Still in many quarters Keralese contributions go unnoticed, D Almeida, J John and A Zadorozhnyy note that a well known historian of mathematics makes:

...No acknowledgement of the work of the Keralese school. [DA/JJ/AZ1, P 78]

(Despite several Western publications of Keralese work.)

Possible Transmission of Keralese Mathematics to Europe

In addition to my discussion, there is a very recent paper (written by D Almeida, J John and A Zadorozhnyy) of great interest, which goes as far as to suggest Keralese mathematics may have been transmitted to Europe. It is true that Kerala was in continuous contact with China, Arabia, and at the turn of the 16th century, Europe, thus transmission might well have been possible. However the current theory is that Keralese calculus remained localised until its discovery by Charles Whish in the late 19th century. There is no evidence of direct transmission by way of relevant manuscripts but there is evidence of methodological similarities, communication routes and a suitable chronology for transmission.

A key development of pre-calculus Europe, that of generalisation on the basis of induction, has deep methodological similarities with the corresponding Kerala development (200 years before). There is further evidence that John Wallis (1665) gave a recurrence relation and proof of Pythagoras theorem exactly as Bhaskara II did. The only

way European scholars at this time could have been aware of the work of Bhaskara would have been through Keralese 'routes'.

The need for greater calendar accuracy and inadequacies in sea navigation techniques are thought to have led Europeans to seek knowledge from their colonies throughout the 16th and 17th centuries. The requirements of calendar reform were imperative with the dating of Easter proving extremely problematic, by the 16th century the European 'Julian' calendar was becoming so inaccurate that without correction Easter would eventually take place in summer! There were significant financial rewards for 'anyone' who could 'assist' in the improvement of navigation techniques. It is thought 'information' was sought from India in particular due to the influence of 11th century Arabic translations of earlier Indian navigational methods.

Events also suggest it is quite possible that Jesuits (Christian missionaries) in Kerala were 'encouraged' to acquire mathematical knowledge while there.

It is feasible that these observations are mere coincidence but if indeed it is true that transmission of ideas and results between Europe and Kerala occurred, then the 'role' of later Indian mathematics is even more important than previously thought.

Conclusions

I wish to conclude initially by simply saying that the work of Indian mathematicians has been severely neglected by western historians, although the situation is improving somewhat. What I primarily wished to tackle was to answer two questions, firstly, why have Indian works been neglected, that is, what appears to have been the motivations and aims of scholars who have contributed to the Eurocentric view of mathematical history. This leads to the secondary question, why should this neglect be considered a great injustice.

I have attempted to answer this by providing a detailed investigation (and analysis) of many of the key contributions of the Indian subcontinent, and where possible, demonstrate how they pre-date European works (whether ancient Greek or later renaissance). I have further developed this 'answer' by providing significant evidence that a number of Indian works conversely influenced later European works, by way of Arabic transmissions. I have also included a discussion of the Indian decimal place value system which is undoubtedly the single greatest Indian contribution to the development of mathematics, and its wider applications in science, economics (and so on).

Discussing my first 'question' is less easy, as within the history of mathematics we find a variety of 'stances'. If the most extreme Eurocentric model is 'followed' then all mathematics is considered European, and even less extreme stances do not give full credit to non-European contributions.

Indeed even in the very latest mathematics histories Indian 'sections' are still generally fairly brief. Why this attitude exists seems to be a cultural issue as much as anything. I feel it important not to be controversial or sweeping, but it is likely European scholars are resistant due to the way in which the inclusion of non-European, including Indian, contributions shakes up views that have been held for hundreds of years, and challenges the very foundations of the Eurocentric ideology. Perhaps what I am trying to say is that prior to discoveries made in technically fairly recent times, and in some cases actually recent times (say in the case of Kerala mathematics) it was generally believed that all science had been developed in Europe. It is almost more in the realms of psychology and culture that we argue about the effect the discoveries of non-European science may have had on the 'psyche' of European scholars.

However I believe this concept of 'late discoveries' is a relatively weak excuse, as there is substantial evidence that

many European scholars were aware of some Indian works that had been translated into Latin. All that aside, there was significant resistance to scientific learning in its totality in Europe until at least the 14th/15th c and as a result, even though Spain is in Europe, there was little progression of Arabic mathematics throughout the rest of Europe during the Arab period.

However, following this period it seems likely Latin translations of Indian and Arabic works will have had an influence. It is possible that the scholars using them did not know the origin of these works. There has also been occasional evidence of European scholars taking results from Indian or Arabic works and presenting them as their own. Actions of this nature highlight the unscrupulous character of some European scholars.

Along with cultural reasons there are no doubt religious reasons for the neglect of Indian mathematics, indeed it was the power of the Christian church that contributed to the stagnation of learning, described as the dark ages, in Europe.

Above all, and regardless of the arguments, the simple fact is that many of the key results of mathematics, some of which are at the very 'core' of modern day mathematics, are of Indian origin. The results were almost all independently 'rediscovered' by European scholars during and after the 'renaissance' and while remarkable, history is something that should be complete and to neglect facts is both ignorant and arrogant. Indeed the neglect of Indian mathematical developments by many European scholars highlights what I can best describe as an idea of European "self importance".

In many ways the results of the Indians were even more remarkable because they occurred so much earlier, that is, advanced mathematical ideas were developed by peoples considered less culturally and academically advanced than (late medieval) Europeans. Although this comment is

controversial it may have been the motivation of several authors for neglect of Indian works, however, if this is the case, then opinions based on those attitudes should be ignored. Indian culture was of the highest standard, and this is reflected in the works that were produced.

Indian mathematicians made great strides in developing arithmetic (they can generally be credited with perfecting use of the operators), algebra (before Arab scholars), geometry (independent of the Greeks), and infinite series expansions and calculus (attributed to 17th/18th century European scholars). Also Indian works, through a variety of translations, have had significant influence throughout the world, from China, throughout the Arab Empire, and ultimately Europe.

To summarise, the main reasons for the neglect of Indian mathematics seem to be religious, cultural and psychological. Primarily it is because of an ideological choice. R Rashed mentions a concept of modernism vs. tradition. Furthermore Indian mathematics is criticised because it lacks rigour and is only interested in practical aims (which we know to be incorrect). Ultimately it is fundamentally important for historians to be neutral, (that includes Indian historians who may go too far the 'other way') and this has not always been the case, and indeed seems to still persist in some quarters.

In terms of consequences of the Eurocentric stance, it has undoubtedly resulted in a cultural divide and 'angered' non-Europeans scholars. There is an unhealthy air of European superiority, which is potentially quite politically dangerous, and scientifically unproductive.

In order to maximise our knowledge of mathematics we must recognise many more nations as being able to provide valuable input, this statement is also relevant to past works. Eurocentrism has led to an historical 'imbalance', which basically means scholars are not presenting an accurate

version of the history of the subject, which I view as unacceptable. Furthermore, it is vital to point out that European colonisation of India most certainly had an extremely negative effect on the progress of indigenous Indian science.

At the very least it must be hoped that the history of Indian mathematics will, in time become as highly regarded, as I believe it should. As D Almeida, J John and A Zadorozhny comment:

...Awareness is not widespread. [DA/JJ/AZ1, P 78]

R Rashed meanwhile explains the current problem:

...The same representation is found time and again: classical science, both in modernity and historicity appears in the final count as work of European humanity alone...

He continues:

...It is true that the existence of some scientific activity in other cultures is occasionally acknowledged. Nevertheless, it remains outside history or is only integrated in so far as it contributed to science, which is essentially European. [RR, P 333]

In short, the doctrine of the western essence of classical science does not take objective history into account.

Finally, beyond simply alerting people to the remarkable developments of Indian mathematicians between around 3000 BC and 1600 AD, and challenging the Eurocentric ideology of the history of the subject, it is thought further analysis and research could also have important consequences for future developments of the subject.

It is thought analysis of the difference in the epistemologies of 17th century European and 15th century Keralese calculus could help to provide an answer to the controversial issue of whether mathematics should concern

itself with proof or calculation. Furthermore, in terms of the way mathematics is currently 'taught' D Almeida, J John and A Zadorozhnyy elucidate:

...The floating point numbers were used by Kerala mathematicians and, using this system of numbers, they were able to investigate and rationalise about the convergence of series. So we (DA/JJ/AZ) believe that a study of Keralese calculus will provide insights into computer-assisted teaching strategies. [DA/JJ/AZ, P 96]

(N.B. computers use a floating-point number system.)

Clearly there is massive scope for further study in the area of the history of Indian and other non-European mathematics, and it is still a topic on which relatively few works have been written, although slowly significantly more attention is being paid to the contributions of non-European countries.

In specific reference to my own project, I would have liked to have been able to go into more depth in my discussion of Indian algebra, and given many more worked examples, as I consider Indian algebra to be both remarkable and severely neglected. Furthermore there is scope for significant and important study of the transmission of Indian mathematics across the world, especially into Europe, via Arabic and later Keralese routes. It is clear that there are many more discoveries to be made and much more that can be written, as C Srinivasiengar observes:

...The last word on the history of ancient civilisation will never be said. [CS, P 1]

As a final note, many question the worth of historical study, beyond personal interest, but I hope I have shown in the course of my work some of the value and importance of historical study. I will conclude with a quote from the scholar G Miller, who commented:

...The history of mathematics is the only one of the sciences to possess a considerable body of perfect and inspiring results which were proved 2000 years ago by the same thought processes as are used today. This history is therefore useful for directing attention to the permanent value of scientific achievements and the great intellectual heritage, which these achievements present, to the world. [AA'D, P 11]

Contribution to Present Day Education

The system of Vedic Mathematics offers a new approach to mathematics and mathematics education. This system which is beautifully coherent and direct was rediscovered from ancient Sanskrit texts between 1911 and 1918 by Bharati Krsna Tirthaji (1884-1960). He found that all problems in pure and applied mathematics can be solved easily with the aid of sixteen Sutras, or word-formulae: for example By One More than the One Before, or Vertically and Crosswise. This may sound incredible but the methods described in his book "Vedic Mathematics" are so original and yet so simple, offering a very different approach to mathematics that is both powerful and fun.

The word-formulae give a set of natural principles that help to quickly solve all sorts of mathematical problems. The Vedic methods are direct, beautifully interrelated and flexible. The Vedic system is therefore much more unified and flowing than conventional mathematics. Some of the methods are truly amazing in their efficiency and simplicity.

The Vedic system is so easy it is really a system of mental mathematics and this, combined with the coherence and flexibility of the system, encourages the development and use of intuition and creativity.

Mental Mathematics

We all make mental calculations from time to time, though we may not always be aware of it. In deciding at

exactly what moment and speed to venture across a busy road, for example, our mind judges continuously the positions and speeds of several vehicles and accurately finds the required gap in which to move forward.

If our mind can make such complex judgements as this it is certainly able to manipulate a few figures. It is the cumbersome calculating devices we have probably been taught- which require pencil and paper or calculator to work out because of their difficulty- and a lack of encouragement for mental calculation which have prevented us from becoming efficient mental calculators.

This need not be so- as Vedic Mathematics shows, mental calculation is easy and to be preferred to pencil and paper or calculator, and has many advantages over these calculating methods.

Most people would probably agree that mathematics holds a special position among subjects of study: that it possess qualities of absolute certainty and precision which cannot be attributed to any other subject. On the other hand however mathematics is seen as difficult and remote by most people- the same people who are also aware of its special absolute qualities. This situation has come about because mathematics education has not been effective enough in bringing out the real nature of mathematics. As young students we glimpse the beauty of mathematics but this is usually a passing phenomenon.

Though mathematics has applications at many levels it is primarily a mental subject. This being so it is likely that lack of mental calculation is partly responsible for the situation described above, and that a system of mental mathematics could provide students with a lasting link with the realm of mathematics and also engender a deeper understanding of the structure and processes of mathematics, as well as helping to develop other important personal qualities.

The following points outline the benefits available from a mental approach to mathematics.

1. Mental calculation sharpens the mind and increases mental agility and intelligence. This will be evident to anyone who has practised or taught mental calculation or who has seen its effects.
2. It enhances the precision of thought. Numbers and other mathematical objects are unbiased, giving only one correct answer to which everyone will agree—there is never a contradiction. This absolute precision is unique to mathematics, so dealing intimately with numbers as we do in mental calculation we cultivate fine and careful thinking.
3. Mental calculation leads naturally to the search for, and discernment of, constancy and law, which are very necessary attributes in a swiftly changing world.
4. Our mind has the ability to retain several ideas at once so that they can be compared, combined and so on. This facility is enhanced by mental calculation as we practise holding the sum in the mind whilst operating with some of the figures.
5. Mental calculation improves the memory. Memory depreciates if it is not exercised. Short term, medium term and long term memory are all stimulated by mental calculation.
6. Because numbers are absolutely dependable and reliable, calculation promotes confidence. In particular, mental calculation creates confidence in oneself and in one's capabilities. To solve a problem, perhaps a difficult one, by mere mental arithmetic without having to rely on some artificial aid is a source of great satisfaction and encouragement.
7. Mental calculation is a delight to the mind: the intrinsic qualities, relationships and beauty of

numbers and the way they create new numbers out of themselves is a source of great enjoyment.

8. Through mental calculation one becomes familiar with numbers and appreciates their various properties. This leads to a real understanding of number.
9. In calculating mentally the subtle properties of numbers and their relationships are appreciated much more readily than if the calculation was written down and thereby fixed. Thus mental calculation leads naturally to innovation and to the invention of new methods, thereby developing the student's natural creativity.
10. Practical uses of mental calculation are many, since we all need to make quick, on the spot, calculations from time to time.

Thus we see that mental calculation has so many advantages and really brings mathematics to life as well as providing motivation and strengthening and enlivening the mind. This is because numbers are mental concepts, they do not exist on paper. Our unhindered mind operates very fast and has a variety of operational properties. With proper training we can use these properties of the mind to our advantage.

This is not to say that pencil and paper or calculating instruments are to be totally avoided in mathematics: they certainly have their place, but mental calculation should, it is suggested, be the primary method of calculation.

India and the World of Mathematics

Secular Education and the Federal Polity

Let me begin by asserting that those who do not understand the past, or refuse to understand it, invariably end up by misunderstanding the present and are unable to move forward into the future. We are faced today with the makers of educational policy in the central government who seem not to understand the Indian past.

There is a constant harking back to a remote past, encapsulated in the phrase 'Vedic'. Irrespective of its historical or civilisational authenticity, this capsule is being forced upon us with the claim that all knowledge is contained in the Vedas and therefore the Vedic capsule amounts to a total education.

There is little recognition of the fact that in the course of Indian history various Indian thinkers discussed the knowledge contained in the Vedic corpus, and some had doubts about various aspects. This process of debate and questioning, the presentation of views and counter-views, both within India and among scholars from other parts of Asia, has been at the root of advances in knowledge in pre-modern times. Much that we pride ourselves on, as Indian contributions to world civilisation, often developed

independently of the Vedic corpus and occasionally even in opposition to it. Significant contributions from the past are thus set aside in this obsessive concern with the Vedic capsule.

In saying this, I am not denigrating the study of the Vedic past, but am emphasising that the past has to be assessed in a historical context, and I would further insist that the context has to be, that of critical, rational enquiry. This is now being denied by replacing enquiry with a received version of the past which is then treated as the authentic version.

The claim is made that this is a return to indigenous knowledge, but the new educational curriculum draws its legitimacy from 19th century colonial views of India, and from the priority that European Indologists gave to brahmanical texts and world-view. Indigenous systems drew not only on mainstream texts in the language of learning but also on texts in a variety of regional languages, which could question the former if need be, as also on observed knowledge.

A major pedagogical change in the last few decades has been the professionalising of various subjects, particularly in the social sciences. Each subject is preferably taught in such a way that it also demonstrates its own methodology which draws as much as possible on evidence of proven reliability, on logical analysis and on rational generalisations. This demands an intellectual rigour in setting out the structure of the subject. The training that results from such teaching, as for example in history, enables both the teacher and the student to be aware of the difference between mythology and history.

There is now a retreat from these processes and mythology is taking over from knowledge. Mythology has a role in creative imagination but should not replace knowledge. Instead of further professionalising the subjects taught at school and college, they are being replaced with subjects

that have virtually no pedagogical rigour, such as Yoga and Consciousness or cultivating a Spirituality Quotient. These cannot form the core of knowledge and replace subjects with a pedagogical foundation, although Yoga can be an additional activity.

The narrowing of knowledge is being attempted in part by giving a single definition to Indian culture and society, and projecting this through educational channels, and describing it as the sole heritage that is of any consequence to us as a society and a nation. Yet this goes against one of the fundamental concerns of the Indian experience both of the past and of the present.

Among the more significant questions that have continually been at the core of Indian activity, is that of the relations between the needs of the central power in a state and the articulation of variant forms of control manifested by regional and local powers. At the most obvious level in the past, this relationship determined various structures relating to administrative and economic policy. But it is also evident in cultural expression where a distinction was often maintained between the mainstream culture, and the culture and language of the region.

Relations between the two varied from close interlinks on some occasions, to tensions or even confrontations on other occasions. What is relevant to us today is that in the past, cooperation between the centre and the regions needed an immense degree of sensitivity to social and cultural variations and an understanding of why they arose. We are facing a similar problem today. The question is whether we should accept the kind of homogenisation of education and culture that is being imposed on the country or should we attempt to define the modern, educated Indian through an educational policy sensitive to a range of social and economic concerns, and to new systems of knowledge, a sensitivity that will provide us with a worthwhile present and enable

us to perceive the inter-connections with the past? Can the interface between the centre and the states in a federal polity, help us in this matter?

Education is not merely about making millions literate, it also involves teaching young people to cope with a changing society, which today means being more aware of the world than ever before, and to creating a worthwhile life for themselves. Therefore, to impose a syllabus oriented to studying an imagined past utopia is to erode the potential of the next generation. Focusing on a utopian past is also a mechanism of diverting attention from having to improve the present in order to provide a better quality of life.

Accountability to the public and transparency in governance is necessary in formulating educational policies. We must know who is drafting educational policy and who have been consulted in doing so, and what has been the participation of professionally qualified persons in the determining of the curriculum in a subject. It requires responsible people and these in turn have to be responsible for what they are doing. Educational policy is both important and sensitive and cannot be left to the whims of a small circle of politicians and bureaucrats.

A sensitive understanding of the interface of centre and region is essential to any educational policy. Two states with high rates of literacy are Kerala and Himachal Pradesh. Each is very different from the other in terms of economic resources and the way they are used; in the hierarchy of castes and the distribution of classes; in religions and religious sects; and in languages. These aspects also undergo change. Can we set aside all this and merely insist on children in both areas studying Sanskrit, Vedic Mathematics, a vague subject called social science, and Yoga and Consciousness? The imposition of the Vedic capsule would be an educational disruption in both regions, educationally negative for many people and resented by others.

But what they do have in common are the aspirations that result from education. Schooling and curriculum would have to relate up to a point to the local conditions and ethos, and these would involve a degree of interest in regional concerns. The question is how best these can be introduced without denying the importance of national concerns - a matter of some sensitivity. Educational policy has to be such that the aspirations at least of regional concerns are recognised as an intrinsic part of those that are of national interest. This would ultimately be more viable than forcing everyone to conform to a top-down policy.

Educational policies in states that do not have a BJP government have a greater responsibility to defend secular education and the continuance of multiple cultures. This is often easier at the state level where multiple cultures are more visible, but would require considerable thinking about education in terms of what is being taught and which groups are appropriating educational facilities. Where parties not belonging to the NDA, tie-up with the Sangh Parivar to harass those supporting secular education, the acts of such parties should also be questioned. Education should not be made the scapegoat for dubious political manoeuvres.

We may well be taking a risk with the future of the next generation by giving them the type of schooling that will not equip them to handle the complexities of our times. These are serious matters that concern the future of an entire generation of young Indians and should be critically discussed and reviewed. But then the Indian middle class is notoriously unconcerned about what it taught to its children through schooling. All that matters is the game of numbers, marks and percentages.

The new policy it is said will reduce social disabilities and the replacing of subjects at school will reduce the burden on the child. Social disabilities can be met to some extent by professionalising what is taught, in other words teaching

mainstream subjects as systems of knowledge, without mystifications. The way a subject is taught has a social context and this has a bearing on social disabilities. For example, will Vedic mathematics be taught through memorising shlokas in Sanskrit or essentially as methods of calculation? In the former case, obviously upper caste children will have an advantage; in case of the latter, the quality of what is taught will have to be assessed comparatively with other mathematical methods. If it were to be something more than a slogan, would this kind of mathematics prepare a foundation for the child to handle contemporary technologies requiring mathematics?

Education is related to the social structures that it endorses or wishes to change. The suggested curriculum is essentially intended to construct a middle class ideology producing pliant citizens. The problem will however be intensified when this regimen is imposed on the underprivileged, and when they become aware of this package being a further denial of quality education to them. With this kind of schooling, dalits and scheduled tribes and other marginalised groups such as women will continue to be employed only for lower level jobs. This may be the other agenda of the new curriculum. The package makes no concession in the curriculum to societies that function differently and require adjustments to educational programmes, such as a greater emphasis initially on technical courses.

Those who are privileged and can afford private schools will continue to have a quality education, but the rest, in terms of educational requirements today, will remain virtually uneducated. Such a two-tier system is implicit in the suggestions on educational programmes made by sections of the corporate sector as well, as was discussed on the first day of the conference. This is likely to be a bigger problem in schools run by individual states and at local levels, since the disaffection will be closer to the ground reality in these

schools. Yet the curriculum in the states can explore the links between education and the social context more closely, as also the links to aspirations through education. This does not mean a tailor-made education for each caste, religious sect, or tribe, but can consider incorporating variations in the handling of knowledge.

The question of the preferred language for education poses other problems. There is today a relationship between English and the regional languages, between Hindi and English, and among the regional languages. Regional languages have become basic to the administration of the states. But given the aspirations to managerial jobs, corporate sector jobs, technologies of various kinds or employment outside India, English is likely to remain in demand. The greater the investment of the multinationals, the greater will be the requirement for professional English. Economic policies could well dictate educational demands giving rise to yet other problems.

A current misconception lies in the belief that education in the medium of English, or an education in the sciences, automatically results in rational attitudes. But rationality has to be cultivated. Rationality is necessary for the study of technologies, for instance, but those who use it for this purpose do not necessarily apply it to understanding other aspects of knowledge. Those that theoretically should be more enlightened often support outdated views about Indian society without a second thought. Education in the sciences and technologies, is divorced from other areas of thought, and often results in a dichotomy between scientific and technological knowledge on the one hand and knowledge related to the social sciences and the humanities on the other. The necessary integration between the two is frequently absent.

An interesting case in point comes from the Indian diaspora. NRIs often demonstrate this dichotomy, where

despite having been trained to enquire critically and rationally into professional areas of work, they make uninformed generalisations about other areas in which they are not trained, such as the Indian past and what goes under the label of Indian culture. This is startlingly evident from their websites. Some of the most untenable theories about the past come from Indian computer scientists, engineers, and such like, and some also claim that because they are scientists, their theories about the past are value-free, a claim that is now much doubted even in scientific circles. Such persons are among the role-models of the Indian middle class. We give legitimacy to their obscurantism because they are technically qualified in the sciences. The reasons for this dichotomy are often explained as due to their lack of adjustment to an alien culture or from the consciousness of being a minority in that culture. If we are now going to have university courses in Vedic rituals, to help create cultural support for such NRIs, then surely we have to concede that the tail is wagging the dog.

If the essentials of critical and rational enquiry in the sciences could be part of a comparative interface with methods in the social sciences, this would result in a far greater awareness of what it means to be educated and how one looks at the world. One is not suggesting that children have to be tutored in the nature of paradigm shifts in all kinds of knowledge, but at least they should know that knowledge changes, that some knowledge becomes out-of-date, and that there are ways of recognising the change. Ultimately the debate between orthodoxies and heterodoxies is essential to advancing knowledge. This debate is a lynchpin in education. The learning process is not a matter of memorising questions and answers but of being encouraged to explore ideas creatively.

To say that educational policy and curriculum should legitimately be replaced with every new government is to make a mockery of the process of education. It is not the

monopoly of any government; it is the responsibility of those who are involved in educating the young. What would be useful would be to work out an essential educational structure, incorporating the major debates in various disciplines, and these should not be tampered with whenever there is a change of government. Changing the content of subjects must be recognised as being a professional matter and not a matter of political ideology.

Education widens the mind and encourages an exploration of ideas and actions. But if it is confined to thinking along a single track, then it closes the mind. In the wider context of planning education, if the direction of change points to a federal polity, then the interface between the centre and the states relates not just to education but to many activities, although educational policies often highlight the nature of the connections. If a federal polity is what is required it cannot be brought about accidentally; it has to be forged. Hopefully, this will be a process that strengthens an inclusive nationalism, secular in its essentials and drawing on creative inter-relations between the centre and the states.

From Seattle to Genoa

AS a reporter who has witnessed many anti-globalization protests, from Seattle in 1999 to Washington D.C., to Prague in September 2000, the police brutality seen in Genoa has no equal so far. True, in Seattle hundreds of youngsters were arrested and in Prague the Czech police deployed tanks around the IMF-World Bank meeting (the last time tanks were seen in the streets of Prague was in 1968, during the Soviet invasion).

The police behaviour in Genoa suggests two questions: one is the accountability of security forces in a democratic country. It is useful to remember that reports by human rights groups such as Amnesty International or Human Rights Watch are replete with accounts of violence and harassments by security forces almost everywhere.

The second question is very Italian, but should sound alarm-bells everywhere: a prime minister who has a virtual monopoly over the electronic media in the country has a political ally in the former fascist party, Alleanza Nazionale (National Alliance).

The vice-premier and leader of Alleanza Nazionale was in Genoa to oversee the security operations. This raises questions about democratic control over the security forces. It was a clear announcement of how social movements will be dealt with in this country. The focus on the violent confrontation diverted the public (and the media) from the 'real' news: with 300,000 people, the Genoa mobilization was second only to the Seattle rally against the WTO and its 'millennium round' of negotiations on trade.

Let's look at who was there: Attac, the Association for a global tax on financial transactions (the Tobin tax), came with thousands of activists from France, Italy, Belgium, even Tunisia. There was the French rural trade union Confederation Paysanne, with its leader Jose Bove, who gained international fame in the summer of 1998 when he destroyed a McDonald's restaurant in a small French town - highlighting the commercial war between Europe and the United States, an example of a new WTO-style trade war (the European Union will not import bovine meat produced in the US and fed with a hormone for animal growth that Europeans consider unsafe for human health; the WTO considers this precautionary principle illegal and sanctioned the EU; the US slammed import fees on a number of European imports like Parma Ham and French cheese).

Environmentalists took to the streets in big numbers, from Green peace to the WWF, to the Italian Legambiente (League for Environment). Many came in small, spontaneous groups. The list includes many Catholic NGOs and the many groups of the Lilliput Network, the 'network of the small people' dealing with fair trade or Third World debt. The

League Against Aids (Lila) displayed banners, a reminder of the legal victory of South Africa against 39 pharmaceutical transnationals on drugs and patents.

There were a multitude of 'affinity groups', heirs of the pacifist movements that in the '80s marched or squatted on military sites against the deployment of US nuclear missiles in Italy, the 'Euromissiles' — a reminder of the Cold War. A number of people marched under the banner of the Italian Metal Workers Federation (FLM): it is the first time a big Italian trade union felt compelled to throw its weight behind a global movement.

In Seattle the mobilization against the WTO was largely fed by the American trade unions Afl-Cio, which organized transport to bring American workers to that north-western town. Their main demand was that trade treaties should include clauses on labour protection (and environmental protection, to please the strong environmentalist and consumer movements that also turned out in Seattle: from the Sierra Club to Public Citizen, the consumer watchdog founded in the '70s by Ralph Nader).

What does all this point to? One could point out that NGOs and movements from the South of the world are under represented in these gatherings. True: in Seattle some Asian and Latin American movements sent delegations, but in Prague or in Genoa there were only a few guests to the Public Forums (the Movimiento Sem Terra, the Brazilian movement of the Peasants Without Land; the Ogoni movement of the Niger Delta, an Indonesian environmental NGO, a representative from an indigenous movement in Colombia or from the Bangkok-based think-tank, Focus on the Global South).

But it was the opposite in Porto Alegre, Brazil, in January 2001, when the World Social Forum gathered to discuss experiences and alternatives (the slogan then was 'Another world is possible').

One could also underline that such diversified gatherings bring together different - sometime conflicting - interests and priorities. So for instance when the American Afl-Cio asks for environmental and labour standard clauses in the trade pacts, the Asian NGOs frown, and with reason. A western-style environmentalism took time to notice that poverty and an unequal share of natural resources are the priority (the climate change negotiation is an example). True: with time, links have been forged between, for instance, movements of indigenous people poisoned by Shell in the Niger Delta and environmental groups in London, or social NGOs in the Amazon and environmental lawyers in Washington, so that common actions can be planned.

The main common point in such a diverse patchwork is probably a crisis of legitimacy of a group of 'most industrialized' governments, the lack of accountability of global institutions like the IMF-World Bank and the WTO, their role in pushing unbridled freedom for financial capital in the name of integrating the global economy, and the dismantling of all regulatory mechanisms (both in the North and in the South) in the name of free trade.

The crisis of legitimacy is matched by a deep ecological crisis, and with the crisis of democracy, together with the new 'logic of violence' created by migrations and the re-definition of cultures and people, as Arjun Appadurai analyzes so convincingly in a recent issue of *Seminar* on Globalization.

Perhaps the economist and director of Focus on the Global South, Walden Bello is right: 'The global economy is so unstable under the menace of so many different crisis, that more and more people from all kinds of backgrounds are directly affected. Of course, each has different reasons to join the protests. ...Now all these different people, with different interests and priorities, are in the process of discovering what they have in common.'

Mathematics appears to have acquired an identity as an independent branch of knowledge early on in human intellectual history. This identity became precise and firmly established thanks to the Greeks in the millennium before Christ. Two characteristics are vital to this identity: abstraction and logical deduction; these are of course present in all scientific enquiries but in mathematics they are defining elements.

Abstraction consists in building mental constructs (often, though not always, born of attempts to understand the concrete). This involves the recognition of common patterns in apparently disparate phenomena and, equally important, the rejection of the irrelevant in an investigation. This is already manifest in the first intimations of mathematical activity: counting. Though almost an involuntary act, what underlies it is profoundly abstract: the human mind recognises that there is an attribute which it can ascribe to a collection of entities - the number of entities in the collection - an attribute which is entirely indifferent to the nature of the individual members of the collection yet can be common to different collections.

What underlies it may be remarkable abstraction, but counting owes its discovery to mundane down to earth compulsions: exchange of goods and barter which required setting values on different commodities. The marketplace was the driving force behind all the arithmetic we learnt at school. And down the ages a great deal of mathematics was born as a response to the needs of diverse human endeavours and has served the cause of these endeavours admirably. This symbiotic relationship is most striking in the case of physics. Attempts to understand motion gave birth to calculus, which has gone on to enhance our understanding not only of motion, but a myriad other natural phenomena in diverse scientific disciplines. The success of mathematics has been so striking that mathematical intervention is what seems to confer the label 'science' on an intellectual discipline.

Mathematics is itself of course a science, yet it stands a little apart from other sciences. The title of Newton's classic, *The Mathematical Principles of Natural Philosophy*, itself suggests this. Gauss (one of the greatest mathematicians of all time), called mathematics the queen of sciences; indeed, at times, one finds in mathematics the aloofness of the royal personage.

The queen is whimsical - queens are supposed to be - but not quite as arbitrary as the one in Alice's Wonderland. There is a large body of mathematics that has been created as a result of purely aesthetic impulses internal to the discipline, essentially by the fancy of the mathematician. Mathematicians perceive beauty in purely mathematical constructs and the interrelation among them and probe these simply to savour the intellectual delights they offer. Yet time and again it has turned out that such mathematics has proved itself the right vehicle for apprehending nature.

The renowned physicist Wigner has described this pithily as the 'unreasonable effectiveness of mathematics in the natural sciences'. A striking example of this 'unreasonable effectiveness' is the way Descartes' idea of representing spatial entities by numbers - coordinates - has permeated our thinking in diverse contexts. Descartes' motivation was the renovation of geometry, as far away as can be from the company chairman pondering the profit curve.

In every creative endeavour there is a certain tension between imagination and discipline. In the sciences other than mathematics, the discipline is essentially external: theories about natural phenomena have to be in tune with observations. The mathematician's imagination is unfettered by external considerations; the discipline comes from the demands of logical deduction and (the more intangible) aesthetics. In this, mathematics is closer to art than to other sciences.

The glorious days of Greek mathematics ended in the early centuries after Christ and the centre of gravity of mathematical development shifted to the East. From then on right up to the 12th century India had a dominant role. One of the greatest mathematical achievements of all time - the invention of 'zero' and the place value system for representing numbers - emanated from this country. It is at once an absolutely brilliant piece of abstract mathematics and simultaneously an unmatched practical device indispensable in practically every sphere of human activity. Arguably, human progress owes more to this one discovery than to any other (mathematical) innovation.

The practical importance of the place value system stems from the ease with which it enables one to handle large numbers though at the time of its discovery there was rarely a serious need to deal with large numbers. Ancient India seems, however, to have been obsessed with large numbers - names had been given to powers of ten way beyond the billion (the ninth power of ten). This suggests that it was the fascination with numbers for their own sake rather than practical considerations that was the motive force behind the discovery - another example of the 'unreasonable effectiveness' of mathematics.

Algebra - the mathematics of manipulation of symbols representing variable quantities - also originated from India; and that too is a magnificent intellectual leap. Arab scholars who came into contact with India recognised the strength of Indian mathematics, absorbed and built upon it, contributing many great ideas themselves. Eventually they passed it all onto Europe which in a burst of new energy took the lead.

Great as its contributions were, the East nevertheless had not, in its pursuit of mathematics, enforced the rigorous discipline of the Greeks. Europe restored firmly Euclid's postulational paradigm as the framework for mathematics.

Today we recognise a body of knowledge as mathematical only if it can be fitted into that framework. It means that the body of knowledge is derived from a certain number of postulates (called axioms by Euclid) which are accepted without argument through rigorous reasoning, the methods of deduction themselves being governed by set rules which are also to be regarded as postulates.

An amazingly large part of what we regard as mathematical knowledge does meet even this very rigid criterion with only the axioms needed for the elementary arithmetic of natural numbers (and just a little more) as the foundation. However, this last statement has to be tempered by one fact: many mathematical areas which today can be fitted into this remarkably economical postulational scheme could not meet these stringent demands when they were first apprehended. It is only after Dedekind in the 19th century showed how to extend the number system beyond the rational numbers of elementary arithmetic, with only the axioms needed for elementary arithmetic as the starting point, that calculus could be incorporated into the fold of this scheme. Contemporaries of Leibnitz and Newton (inventors of calculus), however, had no hesitation in hailing the birth of some new and wonderful mathematics; and not merely calculus. A lot of other much less sophisticated mathematics could be derived from the axioms of elementary arithmetic by Euclid's deductive method but only after Dedekind had his say. Of course, Newton and his contemporaries as well as other mathematicians did use the deductive principles à la Euclid but based themselves implicitly on a set of much more elaborate unquestioned assumptions than the simple axioms of arithmetic.

The modern history - that of the last 500 years - of mathematics is necessarily Eurocentric: practically all major developments right up to the early 20th century originated in Europe. Descartes and Newton were to be followed by a string of great names: Euler, Gauss, Riemann, and Poincare

to name a few. There was a steady and bounteous flow of new ideas through the 18th and 19th centuries, ideas from which emerged whole new areas, and this flow turned into a torrent in the 20th century. Gottingen and Paris were at the heart of this mathematical explosion, an explosion which also fuelled the revolution in physics which was taking place at the same time. The United States of America, not yet a mathematical power, was beginning to display its potential. Czarist Russia was also a participant in the mathematical action in Europe; but with the advent of the Soviet Union, the state came up with a deliberate policy for the promotion of mathematics with resounding effect. By the fifties, Gottingen had declined thanks to the Nazis and was yet to recover from the war while Moscow became a formidable centre rivalling Paris and could boast of a galaxy of extraordinarily creative minds. Sadly, that great school has virtually disintegrated along with the political system: most of the outstanding mathematicians having migrated to the West.

The Americans gave relatively little attention to mathematics in the first half of the 20th century, but the Soviet space programme's first Sputnik jolted them from their benign indifference into eager support for mathematics. Through the sixties and seventies and even into the eighties, support for mathematics was available on a very generous scale in the US and this had a tremendous effect. It produced an array of brilliant mathematicians and much of the most exciting mathematical developments. Europe (Russia included) has now ceded its pre-eminence to the US.

What then of our country? Intellectual activity had certainly taken a back seat for centuries in our country (there was, however, some exceptional mathematical progress in Kerala anticipating later work in Europe). The first stirrings after the long period of dormancy are to be seen in the Bengal renaissance of the 19th century. In the beginning of the reawakening it was the pursuit of humanities

that dominated the scene, but in the early 20th century the twin figures of C.V. Raman and Ramanujan blazed new trails in science.

Raman was an outstanding communicator and his leadership provided immense impetus for the development of physics. Mathematics did not have this advantage: Ramanujan's brilliant career was tragically cut short in its prime. Nevertheless, his example inspired many people to pursue mathematics. A career in mathematics was of course unattractive in comparison with many others when viewed in terms of the creature comforts that one could command, but in the first half of the 20th century there was compensation in the kind of respect that learning was accorded.

It must be said that both Raman and Ramanujan received reasonable support from the colonial institutions of that period; in the case of Ramanujan, once the people in positions of power were convinced of his extraordinary talent, they acted with an alacrity that today's bureaucrats would do well to emulate. Of course, Britain was not interested in promoting intellectual activity in this country, but there was some response to sporadic individual achievements. In any event, whatever the rulers thought, Indian society did not have a strong awareness of the importance of science, much less that of mathematics during the colonial days.

With the advent of independence, the national leadership—Jawaharlal Nehru in particular — laid great emphasis on science and propagated the idea of infusing our society with 'scientific temper'. Nehru's vision resulted in the creation of many institutions of scientific research and among them a few which actively promoted mathematics. However, even as there exists a general perception of science as an important human activity, this perception is (understandably) based on the concrete and practical role science plays in industrial development. There is much less

understanding of the civilizational role of fundamental science in general, of mathematics in particular. There is little appreciation of the fact that a great deal of today's applicable knowledge was at some period in the past basic science at its frontiers. This applies to mathematics much more than to other sciences.

The glamour attached to physics, thanks to developments in the field of nuclear energy and more recently to biology because of the recent discoveries in genetics, helps attract public attention to these fields. Chemistry too has areas that have captured the public imagination. The Nobel Prize also helped increase public awareness of the importance of these sciences: the prize is a household word and some knowledge of facts about it is practically statutory requirement for school kids taking part in quiz contests. Mathematics does not have the advantage of being able to project such a glamorous image and Alfred Nobel unfortunately did not consider mathematics as worthy of partaking of his huge legacy.

Most people, otherwise well informed, are not aware of the Fields Medal which identifies superior mathematical achievement even as the Nobel does in other fields, perhaps because of the fact that its monetary value is a pittance in comparison with the Nobel. Since its inception about 50 mathematicians (necessarily under the age of 40) have been awarded the medal and if a school kid happens to know the name of one of these recipients, it is a safe bet to assume that his/her family has personal contacts with a mathematician.

Even many of our teachers in schools and colleges are unaware of the existence of this medal for mathematics. Very recently, the Norwegian government has instituted a prize similar to the Nobel for mathematics. The mathematical community hopes that it will bring the same kind of visibility to mathematics as Nobel has given to other sciences.

There is a general feeling that unlike physics or other sciences, mathematics is a somewhat other worldly pursuit. Few realize that esoteric problems of cosmology pursued by astrophysicists or the frontier areas of particle physics are as meaningful to the practical everyday world as Fermat's last theorem. As a mathematician I have come across many people who wonder what there is left in mathematics to discover, an experience that I am sure the physicist does not share.

Despite this general lack of public awareness about mathematics, there is in this country a feeling that we are very good at mathematics and there is a certain pride in its past achievements. We have no doubt major past contributions to mathematics to our credit, as has already been pointed out. The romantic story of Srinivasa Ramanujan no doubt contributes (very justifiably) to this belief. But there are other dubious claims on behalf of Vedic mathematics which are also taken quite seriously and contribute to this confidence in our mathematical proficiency. And there is this feeling that people like Shakuntala Devi who can perform calculating feats represent superior mathematical talent that strengthen this perception.

Since Ramanujan, there have been some substantial contributions to mathematics from Indians (working in India), but by and large the public or even the scientific community outside of mathematics are not aware of them. These have made the international community to sit up and take notice of us. A major mathematical event, the International Congress of Mathematicians (ICM) has been taking place regularly once every four years since 1950 with ever increasing international participation. This is the occasion when the Fields Medals are awarded.

The meet is built around 200 prestigious invited lectures: some 20 'plenary' one-hour lectures address a general mathematical audience and are given by eminent figures

who have influenced the evolution of different mathematical areas; and the rest, more specialised talks (of 50-minute duration) by other outstanding contributors to diverse fields within mathematics (the numbers mentioned are from recent congresses).

Some idea of the Indian impact on the world of mathematics may be gleaned from the fact that since 1958, in every Congress except in 1986, there has been at least one invited talk by an Indian mathematician working in India. There have been two plenary talks by Indians but both were non-residents; however, on the whole there have been far fewer talks by non-resident Indians than by those working in India. The Fields Medal has so far eluded us though there are instances of people coming close to winning it. In the developing world, India is certainly in the lead, but China seems poised to overtake us.

That is the 'feel good' side of Indian mathematics, but the peaks do not tell the whole story. The general level of mathematics outside a handful of institutions of higher learning is abysmally low. Much of what is passed off as research in our institutions of higher learning is of shamefully poor quality; nor does one find scholarship of note. For example, if one excludes some half a dozen institutions in the country, it will be impossible to find others even distantly familiar with the mathematical areas in which the fifty odd Field Medallists have worked. Much of the peak achievements have in fact come from two institutions: the Tata Institute of Fundamental Research in Mumbai and the Indian Statistical Institute with branches in Kolkata, Delhi and Bangalore.

The present scenario is indeed depressing, and in many ways things are on a further decline. There has been a steady loss of interest in the pursuit of mathematics (in fact, of all basic science) among the young over the last three decades. In the recent past this problem has become even

more acute. This is largely due to socio-economic factors. Career options for mathematics graduates are essentially limited to the academic profession, not attractive in terms of emoluments: a fresh management graduate enters his/her career at a salary at which the academic retires. Unattractive economic status is of course not new to the academic profession, but in an earlier era it commanded considerable respect in our society and that is no longer the case.

In the case of talented youngsters even these factors do not thwart them from pursuing mathematics, but many seek to pursue it in Europe or America. In the advanced countries too there is a decline of interest in mathematics despite career opportunities being available outside the academia. These migrants from India and others like them from the Third World are in fact now the mainstay of American graduate schools, safeguarding the future of mathematics there. One reason of course is that in these countries the academic has a much better socio-economic standing in society than in our country.

The other is the sorry state of most of our institutions of higher learning - an eager youngster cannot get exposure to truly exciting mathematics except in a limited number of places and the best graduate schools we have, while good, are no match for Harvard or Princeton.

Unfortunately, the policies of the state in regard to higher education do not warrant an optimistic prognosis for the future. The state seems to be reducing, even withdrawing completely such support as it provides now for higher education. This is ostensibly because of the contrast with the meagre resources made available for basic education. There can be no two opinions about the need for enhancing several fold the support given to basic education. But this does not mean that the support given by the state to higher education has to be reduced: it is certainly not excessive by itself. However, it is true that most of the support provided for

higher education has ended up as a 'subsidy' for business and industry (in the country as well as abroad): the prestigious institutions in the country (run practically entirely with government support) have provided human resources for them.

On the other hand higher studies that do not cater to the needs of commerce get short shrift. State governments have steadily cut back on funds for universities. Many university departments have unfilled positions which cannot be advertised, as the governments are not willing to release the money needed for the purpose. Colleges are run with teachers hired on the basis of contracts on unattractive, humiliating terms. In such a context, it is hardly surprising that a very large number of our college teachers are less than competent. The problem gets more acute with areas like mathematics where the subject has been for several years a last resort option for students and these are the ones with the requisite 'qualification' to teach the subject.

In the context of the enormous socio-economic problems facing us, a low priority for the promotion of mathematics may appear entirely justified. But this is definitely short sighted. Mathematics has an ever-increasing role in every human activity. As was pointed out earlier, it played a vital role in the advancement of science and technology. Industry in the advanced countries has benefited immensely from the intervention of mathematics of increasing levels of sophistication. Biology and medicine which seemed at one time to be impervious to mathematics, now use some advanced mathematics. The social sciences too are increasingly turning to mathematics for help. Wall Street employs mathematics PhDs routinely.

All this suggests that the promotion of mathematics is vital to our future well-being. There is another intangible important benefit that society derives from mathematics. Mathematics inculcates habits of thought that help promote

scientific temper. There are many initiatives that can be taken to promote mathematics but they will need political will: we need a major overhaul of the educational system and some social engineering as well. It would appear from our track record that there is mathematical talent and undeniably we have an excellent tradition to revive; what is needed is to create the conditions in which that talent can flower.

Despite all the discouraging factors there is a trickle, small though it is, of headstrong youngsters who pursue mathematics and perform at superior levels working in India. This is the one silver lining in the cloud that keeps optimism alive.

We need to put the astrology debate within the broader framework of the Government's education policy in general, writes Supriya Roy Chowdhury.

He is our quaint R. K. Narayan character, located on the dusty road of Malgudi, sporting a forehead full of sacred ash and vermilion, professional equipment laid out on the pavement - a dozen shells, a square piece of cloth with mystic charts on it, a notebook and a bundle of palm-leaf writing. Narayan forgot to add the inevitable parrot, in a small cage, which invariably accompanies the street side astrologer, ubiquitous anywhere in India. Suddenly this harmless figure becomes a symbol of deep ideological contestation, of serious debates as to whether astrology is science, as the modern Indian academician struggles with the notion of the ash-vermilion-parrot combination making its way into the portals of universities as Reader or Professor. What is a meagre living for an ingenuous, street-smart character, or a harmless pastime for some, suddenly has to become a part of our political discourse. This testifies perhaps that the boundaries of our political culture are limitlessly extendable, that the line between sense and senselessness is indeed thin, and more importantly, that power and

intelligence are really at cross-purposes in the present political scenario.

As a recent editorial of *Current Science* puts it, the battle is not between astrology and science. Rather it is between the UGC - proposing to introduce astrology as a subject of scientific study in universities - and the scientific community. The dynamics of this battle say a great deal not only about a political party using state power to inscribe a particular ideology, but being able to completely ignore democratic procedures and norms in order to push this agenda.

Thus we need to put the astrology debate within the broader framework of the Government's education policy in general. The recently- concluded parliamentary debates on this issue highlighted the essentially undemocratic character in which the National Curriculum Framework for School Education (NCF) was pushed through. It was pointed out, first, that the NCF was enacted without the Minister of Human Resource Development convening a meeting of the Central Advisory Board of Education (CABE). The National Policy of Education, 1986, had laid down the centrality of the CABE in all matters pertaining to changes in education policy. The Minister justified the NCF on grounds that it had been considered by the NCERT and approved by State Education Ministers. In reality it appears that during the NCERT's annual general meeting the NCF was a fait accompli and considered as already passed.

This failure to anchor the process of public policy making in the broad domain of public debate is apparent at every stage of the unfolding drama in education. A large number of academicians has written to the Chairman of the UGC, underlining the non-rational and non-scientific foundations of astrology, and have called for the withdrawal of the scheme to introduce it in universities. The ineffectiveness of these measures of protest stands out. At least two universities

have gone ahead with creating astrology departments. More importantly, the UGC chairman has not replied to any of these protest letters.

Ignoring protests in the expectation that they will die down is not an unusual strategy of governance in India. However, the UGC's blanket of silence underlines two things. First, the UGC possibly has very little to offer by way of a substantive defence in the face of the academics' criticism; second, the Government is obviously prepared to shield the UGC chairman's highly inappropriate behaviour in not responding to these criticisms. All of these are unpleasant reminders that the facade of democracy in India conceals a highly undemocratic culture of governance. In a statement released on April 18, more than 100 scientists in top scientific research institutions across the country called for the UGC Chairman's resignation, and for a broad-based movement to press for his dismissal. Expectedly, the Government has done nothing about this.

One may take a moment to look at the Government's broader justificatory framework for introducing courses in Vedic Mathematics and astrology. This framework underlines the need for value-based education, with a stress on our traditional values. The question of values, when anchored in texts such as Vedas and practices such as astrology, obviously refer to values associated with a particular religious tradition, and goes completely against the logic of a multi-religious nation.

But even apart from this fundamental digression from the secular paradigm, it is extremely odd that a Government unashamedly committed to the introduction of a globalised market in the country should at the same time speak of values, traditional or otherwise. The culture that comes with an aggressively consumption oriented, highly elitist, and crudely inegalitarian model of market-based development, in the context of a desperately poor country,

is nothing if not valueless. It is this double-speak that highlights the Government's hypocrisy more than anything else.

But then, as we also know, Governments are responsive and accountable to the extent that they are forced to be. It is interesting that not one of the national scientific academies has taken an official position in opposition to the Government on the astrology issue. Fellows of the Indian Academy of Sciences have written to the President of the Academy urging him to convey the critical views of the Fellows to the UGC Chairman. These initiatives have brought forth no action on the part of the President, not even in the form of a reply to these letters. (The only exception to this pattern of official silence has been an open letter to the UGC Chairman by Prof. G. Srinivasan, in his capacity as President of the Astronomical Society of India as well as President of the Division on Space and High Energy Astrophysics of the International Astronomical Union.) This discrepancy between the clamour of protests within the scientific community and the failure of the Academies to express an official critique of the UGC's astrology agenda is obviously due to the reluctance of the scientific establishment to engage in a substantive dispute with the powers that be.

The scientific establishment's close dependence upon the state is not only for research funding, but also for things like high profile positions of power within the Government's scientific agencies and committees, and similar largesse that the state is in a position to offer. This provides a partial window to the ineffectiveness of the academics' critique of the Government's education policy.

To their credit, the struggle goes on, as far as the academic community is concerned, supported by organisations such as SAHMAT who have initiated a national debate on the general issue of communalisation of education, and sought to engage a broad spectrum of critical citizens on this question. In a

context where many of the pillars of our democracy are crumbling, the value of such critical dissent cannot be emphasized enough.

One wishes, though, that the academic community's critical discourse was not so imprisoned within the immediacy of the astrology/saffronisation issue. Universities in India have been in a state of decline for many years now. The elite of the academic science community, ensconced in the comfort of top research institutions, and inhabiting, for all practical purposes, an international (read western) professional habitat, has rarely, if ever, reacted to the rapid degeneration of the university system. This curiously ostrich-like approach fails to recognise that a declining university system cuts at the roots of higher education in any context, and in fact potentially destroys the foundations on which these elite scientific research institutions stand. Therefore, if the protection of universities and of science education is the broad concern, one needs to acknowledge that saffronisation maybe the biggest threat, but not the only one. There would be a greater measure of credibility, and perhaps power, in the academics' protest if it was anchored within the broad issue of institutional erosion in education.

Criticism

We, the undersigned, are deeply concerned by the continuing attempts to thrust the so-called 'Vedic Mathematics' on the school curriculum by the NCERT. As has been pointed out earlier on several occasions, the so-called 'Vedic Mathematics' is neither 'Vedic' nor can it be dignified by the name of mathematics. 'Vedic Mathematics', as is well-known, originated with a book of the same name by a former Sankracharya of Puri (the late Jagadguru Swami Shri Bharati Krishna Tirthaji Maharaj) published posthumously in 1965. The book assembled a set of tricks in elementary arithmetic and algebra to be applied in

performing computations with numbers and polynomials. As is pointed out even in the Foreword to the book by the General Editor, Dr. A.S. Agarwala, the aphorisms in Sanskrit to be found in the book have nothing to do with the Vedas. Nor are these aphorisms to be found in the genuine Vedic literature. The term "Vedic maths" is therefore entirely misleading and factually incorrect.

Further, it is clear from the notation used in the arithmetical tricks in the book that the methods used in this text have nothing to do with the arithmetical techniques of antiquity. Many of the Sanskrit aphorisms in the book are totally cryptic (ancient Indian mathematical writing was anything but cryptic) and often so general as to be devoid of any specific mathematical meaning. There are several authoritative texts on the mathematics of Vedic times that could be used in part to teach an authoritative and correct account of ancient Indian mathematics but this book clearly cannot be used for any such purpose. The teaching of mathematics involves both the teaching of the basic concepts of the subject as well as methods of mathematical computation. The so-called "Vedic maths" is entirely inadequate to this task considering that it is largely made up of tricks to do some elementary arithmetic computations. Many of these can be far more easily performed on a simple computer or even an advanced calculator. The book "Vedic maths" essentially deals with arithmetic of the middle and high-school level.

Its claims that "there is no part of mathematics, pure or applied, which is beyond their jurisdiction" is simply ridiculous. In an era when the content of mathematics teaching has to be carefully designed to keep pace with the general explosion of knowledge and the needs of other modern professions that use mathematical techniques, the imposition of "Vedic mathematics" will be nothing short of calamitous. India today has active and excellent schools of research and

teaching in mathematics that are at the forefront of modern research in their discipline with some of them recognised as being among the best in the world in their fields of research. It is noteworthy that they have cherished the legacy of distinguished Indian mathematicians like Srinivasa Ramanujam, V. K. Patodi, S. Minakshisundaram, Harish Chandra, K. G. Ramanathan, Hansraj Gupta, Syamdas Mukhopadhyay, Ganesh Prasad, and many others. Including several living Indian mathematicians. But not one of these schools has lent an iota of legitimacy to 'Vedic maths'. Nowhere in the world does any school system teach "Vedic maths" or any form of ancient maths for that matter as an adjunct to modern mathematical teaching. The bulk of such teaching belongs properly to the teaching of history and in particular the teaching of the history of the sciences.

We consider the imposition of 'Vedic maths' by a Government agency, as the perpetration of a fraud on our children, condemning particularly those dependent on public education to a sub-standard mathematical education. Even if we assumed that those who sought to impose 'Vedic maths' did so in good faith, it would have been appropriate that the NCERT seek the assistance of reknowned Indian mathematicians to evaluate so-called "Vedic mathematics" before making it part of the National Curricular framework for School Education.

Appallingly they have not done so. In this context we demand that the NCERT submit the proposal for the introduction of 'Vedic maths' in the school curriculum to recognized bodies of mathematical experts in India, in particular the National Board of Higher Mathematics (under the Dept. of Atomic Energy), and the Mathematics sections of the Indian Academy of Sciences and the Indian National Science Academy, for a thorough and critical examination. In the meanwhile no attempt should be made to thrust the subject into the school curriculum either through the

Centrally-administered school system or by trying to impose it on the school systems of various States.

We are concerned that the essential thrust behind the campaign to introduce the so-called 'Vedic mathematics' has more to do with promoting a particular brand of religious majoritarianism and associated obscurantist ideas rather than any serious and meaningful development of mathematics teaching in India.

We note that similar concerns have been expressed about other aspects too of the National Curricular Framework for School Education. We re-iterate our firm conviction that all teaching and pedagogy, not just the teaching of mathematics, must be founded on rational, scientific and secular principles.

Vedic Mathematics—A Briefing

In the previous chapters we have gone through the Vedic Mathematics Sutras and upa-Sutras: their application in solving problems. In this approach we have missed to note some points and merits of one method over the other methods at some instances.

Now we take a few steps in this direction. You may question why this book first gives examples and methods and then once again try to proceed as if an introduction to the Vedic Mathematics has been just started. This is because in this approach the reader first feels that it is easy to solve problems using Vedic Mathematics.

This is clear from the examples given. But the reader may get doubt why we are doing this way or that way some times very close and almost akin to the conventional textual way; and some times very different from these procedures? why new representations and different meanings for the same Sutra (!) in different contexts? But observe that it is not uncommon to Mathematics.

Question some body showing the symbol Π .

Majority may say it is $22/7$ (is it right?) some may say it is a radian measure. Very few may state it is a function or so.

What does the representation $A \times B$ mean?

A boy thinking about numbers may answer that is A multiplied by B and gives the product provided A and B are known. A girl thinking of set notation simply says that it is Cartesian product of the sets A and B . No sort of multiplication at all.

Another may conclude that it is a product of two matrices A and B . No doubt a multiplication but altogether different from above.

Some other may go deep in to elementary number theory and may take ' X ' to be the symbol ' X ' (does not divide) and conclude ' A does not divide B '.

Now the question arises does a student fail to understand and apply contextual meaning and representation of symbols and such forms in mathematical writings? certainly not. In the same way the contextual meanings of the Sutras also can not bring any problem to the practitioners of Vedic Mathematics.

Again a careful observation brings all of us to a conclusion that even though the Sutras are not like mathematical formulae so as to fit in any context under consideration but they are intended to recognize the pattern in the problems and suggest procedures to solve. Now recall the terms, rules and methods once again to fill in some gaps that occur in the previous attempt.

Terms and Operations

a) *Ekadhika* means 'one more'

e.g: *Ekadhika* of 0 is 1

Ekadhika of 1 is 2

Ekadhika of 8 is 9

Ekadhika of 23 is 24

Ekadhika of 364 is 365-----

- b) Ekanyuna means 'one less'

e.g: Ekanyuna of 1 2 3 8 1469

is 0 1 2 713 68

- c) Purak means 'complement'

e.g: purak of 1 2 3 8., 9 from 10

is 9 8 7 2 1

- d) Rekhank means 'a digit with a bar on its top'. In other words it is a negative number.

e.g: A bar on 7 is $\bar{7}$. It is called rekhank 7 or bar 7.

We treat purak as a Rekhank.

e.g.: $\bar{7}$ is 3 and $\bar{3}$ is 7

At some instances we write negative numbers also with a bar on the top of the numbers as

-4 can be shown as $\bar{4}$.

-21 can be shown as $\bar{21}$.

- e) Addition and subtraction using Rekhank.

Adding a bar-digit i.e. Rekhank to a digit means the digit is subtracted.

e.g.: $3 + \bar{1} = 2$, $5 + \bar{2} = 3$, $4 + \bar{4} = 0$

Subtracting a bar - digit i.e. Rekhank to a digit means the digit is added.

e.g.: $4 - \bar{1} = 5$, $6 - \bar{2} = 8$, $3 - \bar{3} = 6$

1. Some more examples

e.g.: $3 + 4 = 7$

$(-2) + (-5) = \bar{2} + \bar{5} = \bar{7}$ or -7

f) Multiplication and Division using rekhanak.

1. Product of two positive digits or two negative digits (Rekhanaks)

e.g.: $2 \times 4 = 8$; $\bar{4} \times \bar{3} = 12$ i.e. always positive

2. Product of one positive digit and one Rekhanak

e.g.: $3 \times \bar{2} = \bar{6}$ or -6 ; $\bar{5} \times 3 = \bar{15}$ or -15 i.e. always Rekhanak or negative.

3. Division of one positive by another or division of one Rekhanak by another Rekhanak.

e.g.: $8 \div 2 = 4$, $\bar{6} \div \bar{3} = 2$ i.e. always positive

4. Division of a positive by a Rekhanak or vice versa.

e.g.: $\bar{10} \div 5 = \bar{2}$, $6 \div \bar{2} = \bar{3}$ i.e. always negative or Rekhanak.

g) *Beejanak*: The Sum of the digits of a number is called Beejanak. If the addition is a two digit number, Then these two digits are also to be added up to get a single digit.

e.g.: Beejanak of 27 is $2 + 7 = 9$.

Beejanak of 348 is $3 + 4 + 8 = 15$

Further $1 + 5 = 6$. i.e. 6 is Beejanak.

Beejanak of 1567 $\rightarrow 1 + 5 + 6 + 7 \rightarrow 19$ $1 + 9 \rightarrow 1$

i.e. Beejanak of 1567 is 1.

ii) *Easy way of finding Beejanak*: Beejanak is unaffected if 9 is added to or subtracted from the

number. This nature of 9 helps in finding Beejank very quickly, by cancelling 9 or the digits adding to 9 from the number.

e.g. 1: Find the Beejank of 632174.

As above we have to follow

$$632174 \rightarrow 6 + 3 + 2 + 1 + 7 + 4 \rightarrow 23 \quad 2 + 3 \rightarrow 5$$

But a quick look gives 6 & 3 ; 2 & 7 are to be ignored because $6 + 3 = 9$, $2 + 7 = 9$. Hence remaining $1 + 4 \rightarrow 5$ is the beejank of 632174:

e.g. 2:

$$\text{Beejank of } 1256847 \rightarrow 1 + 2 + 5 + 6 + 8 + 4 + 7 \rightarrow 33 \rightarrow 3+3 \rightarrow 6.$$

But we can cancel 1& 8, 2& 7, 5 & 4 because in each case the sum is 9. Hence remaining 6 is the Beejank.

- h) *Check by Beejank Method:* The Vedic sutra - Gunita Samuccayah gives 'the whole product is same'. We apply this sutra in this context as follows. It means that the operations carried out with the numbers have same effect when the same operations are carried out with their Beejanks.

Observe the following examples.

- i) $42 + 39$

Beejanks of 42 and 39 are respectively $4 + 2 = 6$ and $3 + 9 = 12$ and $1 + 2 = 3$

Now $6 + 3 = 9$ is the Beejank of the sum of the two numbers

Further $42 + 39 = 81$. Its Beejank is $8 + 1 = 9$.

We have checked the correctness.

- ii) $64 + 125$.

$$64 \rightarrow 6 + 4 \rightarrow 10 \rightarrow 1 + 0 \rightarrow 1$$

$$125 \rightarrow 1 + 2 + 5 \rightarrow 8$$

Sum of these Beejanks $8 + 1 = 9$

Note that

$$64 + 125 = 189 \rightarrow 1 + 8 + 9 \rightarrow 18 \quad 1 + 8 \rightarrow 9$$

we have checked the correctness.

iii) $134 - 49$

$$134 \rightarrow 1 + 3 + 4 \rightarrow 8$$

$$49 \rightarrow 4 + 9 \rightarrow 13 \rightarrow 1 + 3 \rightarrow 4$$

Difference of Beejanks $8 - 4 \rightarrow 4$,

note that $134 - 49 = 85$

Beejanks of 85 is $8 + 5 \rightarrow 85 \rightarrow 8 + 5 \rightarrow 13 \rightarrow 1 + 3 \rightarrow 4$
verified.

iv) $376 - 284$

$$376 \rightarrow 7 \therefore (6 + 3 \rightarrow 9)$$

$$284 \rightarrow 2 + 8 + 4 \rightarrow 14 \rightarrow 1 + 4 \rightarrow 5$$

Difference of Beejanks $= 7 - 5 = 2$

$$376 - 284 = 92$$

$$\text{Beejank of } 92 \rightarrow 9 + 2 \rightarrow 11 \quad 1 + 1 \rightarrow 2$$

Hence verified.

v) $24 \times 16 = 384$

Multiplication of Beejanks of

$$24 \text{ and } 16 \text{ is } 6 \times 7 = 42 \rightarrow 4 + 2 \rightarrow 6$$

$$\text{Beejank of } 384 \rightarrow 3 + 8 + 4 \rightarrow 15 \quad 1 + 5 \rightarrow 6$$

Hence verified.

vi) $237 \times 18 = 4266$

$$\text{Beejank of } 237 \rightarrow 2 + 3 + 7 \rightarrow 12 \rightarrow 1 + 2 \rightarrow 3$$

$$\text{Beejank of } 18 \rightarrow 1 + 8 \rightarrow 9$$

$$\text{Product of the Beejanks} = 3 \times 9 \rightarrow 27 \rightarrow 2 + 7 \rightarrow 9$$

$$\text{Beejank of } 4266 \rightarrow 4 + 2 + 6 + 6 \rightarrow 18 \rightarrow 1 + 8 \rightarrow 9$$

Hence verified.

vii) $242 = 576$

Beejank of 24 $\rightarrow 2 + 4 \rightarrow 6$

square of it $62 \rightarrow 36 \rightarrow 9$

Beejank of result $= 576 \rightarrow 5 + 7 + 6 \rightarrow 18 \rightarrow 1 + 8 \rightarrow 9$

Hence verified.

viii) $3562 = 126736$

Beejank of 356 $\rightarrow 3 + 5 + 6 \rightarrow 5$

Square of it $= 52 = 25 \rightarrow 2 + 5 \rightarrow 7$

Beejank of result $126736 \rightarrow 1 + 2 + 6 + 7 + 3 + 6 \rightarrow 1 + 6 \rightarrow 7$

$\therefore (7 + 2 = 9; 6 + 3 = 9)$ hence verified.

- ix) *Beejank in Division:* Let P, D, Q and R be respectively the dividend, the divisor, the quotient and the remainder.

Further the relationship between them is $P = (Q \times D) + R$

E.g. 1: $187 \div 5$

we know that $187 = (37 \times 5) + 2$ now the Beejank check.

We know that $187 = (37 \times 5) + 2$ now the Beejank check.

$187 \rightarrow 1 + 8 + 7 \rightarrow 7 \therefore (1 + 8 = 9)$

$(37 \times 5) + 2 \rightarrow$ Beejank $[(3 + 7) \times 5] + 2 \rightarrow 5 + 2 \rightarrow 7$

Hence verified.

E.g. 2: $7986 \div 143$

$7896 = (143 \times 55) + 121$

Beejank of 7986 $\rightarrow 7 + 9 + 8 + 6 \rightarrow 21$

($\therefore 9$ is omitted) $\rightarrow 2 + 1 \rightarrow 3$

Beejank of $143 \times 55 \rightarrow (1 + 4 + 3) (5 + 5)$

$\rightarrow 8 \times 10 \rightarrow 80 (8 + 0) \rightarrow 8$

Beejank of $(143 \times 55) + 121 \rightarrow 8 + (1 + 2 + 1)$
 $\rightarrow 8 + 4 \rightarrow 12 \quad 1 + 2 \rightarrow 3$

Hence verified.

Check the following results by Beejank Method

1. $67 + 34 + 82 = 183$
2. $4381 - 3216 = 1165$
3. $63^2 = 3969$
4. $(1234)^2 = 1522756$
5. $(86 \times 17) + 34 = 1496$
6. $2556 \div 127$ gives $Q = 20, R = 16$

Vinculum: The numbers which by presentation contains both positive and negative digits are called vinculum numbers.

Conversion of general numbers into vinculum numbers.

We obtain them by converting the digits which are 5 and above 5 or less than 5 without changing the value of that number.

Consider a number say 8.

(Note it is greater than 5).

Use its complement (purak - rekhank) from 10. It is 2 in this case and add 1 to the left (i.e. tens place) of 8.

Thus $8 = 08 = 1\bar{2}$.

The number 1 contains both positive and negative digits

i.e. 1 and $\bar{2}$. Here $\bar{2}$ is in unit place hence it is -2 and value of 1 at tens place is 10.

Thus $1\bar{2} = 10 - 2 = 8$

Conveniently we can think and write in the following way:

General Number	Conversion	Vinculum Number
6	10 - 4	$\overline{14}$
97	100 - 3	$10\overline{3}$
289	300 - 11	$10\overline{3}$ etc.

The sutras 'Nikhilam Navatascharamam Dasatah' and 'Ekadhikena purvena' are useful for conversion.

e.g. 1: 289, Edadhika of 2 is 3

Nikhilam from 9 : $8 - 9 = -1$ or $\overline{1}$

Charmam from 10 : $9 - 10 = -1$ or $\overline{1}$

i.e. 289 in vinculum form $\overline{311}$

e.g. 2: 47768

'Ekadhika' of 4 is 5

'Nikhilam' from 9 (of 776) $\overline{223}$

'Charmam' from 10 (of 8) $\overline{2}$

Vinculum of 47168 is 5 $\overline{2232}$

e.g. 3: 11276

Here digits 11 are smaller. We need not convert. Now apply for 276 the two sutras Ekadhika of 2 is 3

'Nikhilam Navata' for 76 is $\overline{24}$

$$11276 = 113\overline{24}$$

i.e. $113\overline{24} = 11300 - 24 = 11276.$

The conversion can also be done by the sutra sankalana vyavakalanabhyam as follows.

e.g. 4: 315

sankalanam (addition) = $315 + 315 = 630$.

Vyvakalanam (subtraction) = $630 - 315 = 325$

Working steps :

$$0 - 5 = \bar{5}$$

$$3 - 1 = 2$$

$$6 - 3 = 3$$

Let's apply this sutra in the already taken example 47768.

Samkalanam = $47768 + 47768 = 95536$

Vyavakalanam = $95536 - 47768$.

$$\begin{array}{r} \text{Steps: } 6-8=\bar{2} \\ 3-6=\bar{3} \\ 5-7=\bar{2} \\ 5-7=\bar{2} \\ 9-4=5 \end{array} \left. \vphantom{\begin{array}{r} 6-8=\bar{2} \\ 3-6=\bar{3} \\ 5-7=\bar{2} \\ 5-7=\bar{2} \\ 9-4=5 \end{array}} \right\} = \overline{52232}$$

Consider the conversion by sankalanavyavakalanabhyam and check it by Ekadhika and Nikhilam.

e.g. 5: 12637

1. Sankalana gives, $12637 + 12637 = 25274$

$$25274 - 12637 = (2 - 1)/(5 - 2)/(2 - 6)/(7 - 3)/(4 - 7) = \overline{13443}$$

2. Ekadhika and Nikhilam gives the following.

As in the number 1 2 6 3 7, the smaller and bigger digits (i.e. less than 5 and; 5, greater than 5) are mixed up, we split up in to groups and conversion is made up as given below.

Split 1 2 6 and 3 7

Now the sutra gives 1 2 6 as $13\bar{4}$ and 37 as $4\bar{3}$

Thus $12637 = 1\bar{3}4\bar{4}3$

Now for the number 315 we have already obtained vinculum as 325 by "sankalana ... " Now by 'Ekadhika and Nikhilam ...' we also get the same answer.

315 Since digits of 31 are less than 5,

We apply the sutras on 15 only as

Ekadhika of 1 is 2 and Charman of 5 is $\bar{5}$.

Consider another number which comes under the split process.

e.g. 6: 24173

As both bigger and smaller numbers are mixed up we split the number 24173 as 24 and 173 and write their vinculum by Ekadhika and Nikhilam sutras as

$$24 = 3\bar{6} \text{ and } 173 = 2\bar{2}7$$

$$\text{Thus } 24173 = 3\bar{6}2\bar{2}7$$

Convert the following numbers into vinculum number by i. Ekadhika and Nikhilam sutras, and ii. Sankalana vyavakalana sutra.

Observe whether in any case they give the same answer or not.

1. 64
2. 289
3. 791
4. 2879
5. 19182
6. 823672
7. 123456799
8. 65384738

Conversion of vinculum number into general numbers.

The process of conversion is exactly reverse to the already done. Rekhanks are converted by Nikhilam where as other digits by 'Ekanyunena' sutra. thus:

$$\begin{aligned} \text{i) } 1\bar{2} &= (1 - 1) / (10 - 2) \text{ Ekanyunena } 1 - 1 \\ &= 08 = 8 \qquad \text{Nikhilam. } 2 = 10 - 2 \end{aligned}$$

$$\begin{aligned} \text{ii) } 3\bar{2}6 &= (3 - 1) / (9 - 2) / (10 - 6) \\ &= 274 \end{aligned}$$

$$\begin{aligned} \text{iii) } 3\bar{3}4\bar{4} &= (3 - 1) / (10 - 3) / (4 - 1) / (10 - 4) \\ &= 2736 \text{ (note the split)} \end{aligned}$$

$$\begin{aligned} \text{iv) } 20\bar{3}40\bar{1}2\bar{1} &= 2/(0-1)/(9-3)/(10-4)/(0-1)/(9-1)/(10-2)/1 \\ &= 2\bar{1}66\bar{1}881 \\ &= 2\bar{1} / 6 / 6\bar{1} / 881. \text{ once again split} \\ &= (2 - 1) / (10 - 1) / 6 / (6 - 1) / (10 - 1) / 881 \\ &= 19659881 \end{aligned}$$

$$\begin{aligned} \text{v) } 30\bar{3}21\bar{2} &= 3 / 0321 / 2 \\ &= 3 / (0-1) / (9-3) / (9-2) / (10-1) / 2 \\ &\quad 3 / \bar{1} / 6792 \\ &\quad (3 - 1) / (10 - 1) / 6792 \\ &= 296792. \end{aligned}$$

Single to many conversions.

It is interesting to observe that the conversions can be shown in many ways.

e.g. 1: 86 can be expressed in following many ways

$$86 = 90 - 4 = \overline{94}$$

$$= 100 - 14 = \overline{114}$$

$$= 1000 - 914 = \overline{1914}$$

$$\text{Thus } 86 = \overline{94} = \overline{114} = \overline{1914} = 19914 = \dots\dots\dots$$

e.g. 2 :

$$\overline{07} = -10 + 3 = \overline{13}$$

$$\overline{36} = -100 + 64 = \overline{164}$$

$$\overline{978} = -1000 + 22 = \overline{1022} \text{ etc.,}$$

*** Convert by Vedic process the following numbers into vinculum numbers.**

- 1) 274
- 2) 4898
- 3) 60725
- 4) 876129.

Convert the following vinculum numbers into general form of numbers (normalized form)

- 1) 283
- 2) 3619
- 3) 27216
- 4) 364718
- 5) 60391874

Vedic check for conversion.

The vedic sutra "Gunita Samuctayah" can be applied for verification of the conversion by Beejank method.

Consider a number and find its Beejank. Find the vinculum number by the conversion and find its Beejank. If both are same the conversion is correct.

e.g..

$$196 = \bar{2}16.$$

Now Beejank of 196 $\rightarrow 1 + 6 \rightarrow 7$

Beejank of 216 $\rightarrow 2 + (-1) + 6 \rightarrow 7$.

Thus verified.

But there are instances at which, if beejank of vinculum number is rekhank i.e. negative. Then it is to be converted to +ve number by adding 9 to Rekhank (already we have practised) and hence 9 is taken as zero, or vice versa in finding Beejank.

e.g.:

$$\bar{2}1\bar{3} = 200 - 13 = 187$$

Now Beejank of $21\bar{3} = 2 + (-1) + (-3) = -2$

Beejank of 187 $= 1 + 8 + 7 = 16 \rightarrow 1 + 6 = 7$

The variation in answers can be easily understood as

$$\bar{2} = \bar{2} + 9 \rightarrow -2 + 9 = 7.$$

Hence verified.

Use Vedic check method of the verification of the following result.

1. $24 = \bar{3}6$

2. $2736 = 3\bar{3}4\bar{4}.$

3. $3\bar{2}6 = 274$

4. $2\bar{3}2\bar{1}3 = 17187$

Addition and Subtraction Using Vinculum Numbers

e.g. 1: Add 7 and 6 i.e., $7 + 6$.

- i) Change the numbers as vinculum numbers as per rules already discussed.

$$\{ 7 = 1\bar{3} \text{ and } 6 = 1\bar{4} \}$$

- ii) Carry out the addition column by column in the normal process, moving from top to bottom or vice versa.
- iii) add the digits of the next higher level i.e., $1 + 1 = 2$

$$1\bar{3}$$

$$1\bar{4}$$

$$2\bar{7}$$

- iv) the obtained answer is to be normalized as per rules already explained. rules already explained.

$$\text{i.e., } 2\bar{7} = (2 - 1)(10 - 7) = 13 \text{ Thus we get } 7 + 6 = 13.$$

e.g. 2 : Add 973 and 866.

$$973 = 10\bar{3}3$$

$$10\bar{3}3$$

$$866 = 11\bar{3}4$$

$$11\bar{3}4$$

$$21\bar{6}1$$

$$\text{But } 21\bar{6}1 = 2000 - 161 = 1839.$$

Thus $973 + 866$ by vinculum method gives 1839 which is correct.

Observe that in this representation the need to carry over from the previous digit to the next higher level is almost not required.

e.g. 3 : Subtract 1828 from 4247.

i.e.,

$$4247$$

$$-1828$$

Step:

- (i) Write -1828 in Bar form i.e., $\overline{1828}$
- (ii) Now we can add 4247 and $\overline{1828}$ i.e.,

$$4247$$

$$+ \overline{1828}$$

$$\hline 3\overline{6}2\overline{1}$$

Since $7 + \overline{8} = \overline{1}$,

$$4 + \overline{2} = 2,$$

$$2 + \overline{8} = \overline{6},$$

$$4 + \overline{1} = 3$$

- (iii) Changing the answer 3621 into normal form using Nikhilam, we get

$$3\overline{6}2\overline{1} = 36 / 21 \text{ split}$$

$$= (3 - 1) / (10 - 6) / (2 - 1) / (10 - 1) = 2419$$

$$\therefore 4247 - 1828 = 2419$$

Find the following results using Vedic methods and check them

1) $284 + 257$

2) $5224 + 6127$

3) $582 - 464$

4) $3804 - 2612$

Addition and Subtraction**Addition**

In the convention process we perform the process as follows.

$$234 + 403 + 564 + 721$$

write as 234

403

564

721

Step (i): $4 + 3 + 4 + 1 = 12$ 2 retained and 1 is carried over to left.

Step (ii): $3 + 0 + 6 + 2 = 11$ the carried '1' is added i.e., Now 2 retained as digit in the second place (from right to left) of the answer and 1 is carried over to left.

Step (iii): $2 + 4 + 5 + 7 = 18$ carried over '1' is added i.e., $18 + 1 = 19$. As the addition process ends, the same 19 is retained in the left hand most part of the answer.

Thus 234

403

564

+721

1922 is the answer

we follow sudhikaran process Recall 'sudha' i.e., dot (.) is taken as an upa-sutra (No: 15)

Consider the same example

234

403

564 ↑

+721 |

(i) Carry out the addition column by column in the usual fashion, moving from bottom to top.

(a) $1 + 4 = 5$, $5 + 3 = 8$, $8 + 4 = 12$ The final result is more than 9. The tenth place '1' is dropped

once number in the unit place i.e., 2 retained. We say at this stage sudha and a dot is above the top 4. Thus column (1) of addition (right to left)

$$\begin{array}{r} . \\ 4 \\ 3 \\ 4 \\ 1 \\ \hline 2 \end{array}$$

- (b) Before coming to column (2) addition, the number of dots are to be counted, This shall be added to the bottom number of column (2) and we proceed as above. Thus second column becomes

$$\begin{array}{r} . \\ 3 \text{ dot} = 1, \quad 1 + 2 = 3 \\ 0 \quad \quad \quad 3 + 6 = 9 \\ 6 \quad \quad \quad 9 + 0 = 9 \\ 2 \quad \quad \quad 9 + 3 = 12 \\ \hline 2 \end{array}$$

2 retained and '.' is placed on top number 3

- (c) proceed as above for column (3)

$$\begin{array}{ll} 2 & \text{i) dot} = 1 \quad \quad \quad \text{ii) } 1 + 7 = 8 \\ 4 & \text{iii) } 8 + 5 = 13 \quad \quad \text{iv) Sudha is said.} \end{array}$$

$$\begin{array}{r} . \\ 5 \\ 7 \\ \hline 9 \end{array} \quad \begin{array}{l} \text{A dot is placed on 5 and proceed} \\ \text{with retained unit place 3.} \end{array}$$

$$\text{v) } 3 + 4 = 7, 7 + 2 = 9$$

Retain 9 in 3rd digit i.e., in 100th place.

- (d) Now the number of dots is counted. Here it is 1 only and the number is carried out left side i.e. 1000th place

$$\begin{array}{r}
 \text{Thus} \quad 234 \\
 \quad \quad 403 \\
 \quad \quad \quad \cdot \\
 \quad \quad 564 \\
 \quad + 721 \\
 \hline
 \end{array}$$

1922 is the answer.

Though it appears to follow the conventional procedure, a careful observation and practice gives its special use.

e.g. (1):

$$\begin{array}{r}
 \cdot \\
 437 \\
 \cdot \cdot \\
 624 \\
 \cdot \\
 586 \\
 + 162 \\
 \hline
 1809
 \end{array}$$

Steps 1:

- (i) $2 + 6 = 8$, $8 + 4 = 12$ so a dot on 4 and $2 + 7 = 9$ the answer retained under column (i)
- (ii) One dot from column (i) treated as 1, is carried over to column (ii), thus $1 + 6 = 7$, $7 + 8 = 15$ A 'dot' is placed on 8 for the 1 in 15 and the 5 in 15 is added to 2 above.
 $5 + 2 = 7$, $7 + 3 = 10$ i.e. 0 is written under column (ii) and a dot for the carried over 1 of 10 is placed on the top of 3.

- (iii) The number of dots counted in column (iii) are 2. Hence the number 2 is carried over to column (ii) Now in column (iii) $2 + 1 = 3$, $3 + 5 = 8$, $8 + 6 = 14$ A dot for 1 on the number 6 and 4 is retained to be added 4 above to give 8. Thus 8 is placed under column (iii).
- (iv) Finally the number of dots in column (iii) are counted. It is '1' only. So it carried over to 1000th place. As there is no fourth column 1 is the answer for 4th column. Thus the answer is 1809.

EXAMPLE 3:

$$\begin{array}{r}
 2\dot{6}4\dot{7} \\
 40\dot{8}6 \\
 5\dot{7}1\dot{8} \\
 +16\dot{9}2 \\
 \hline
 15143
 \end{array}$$

Check the result verify these steps with the procedure mentioned above.

The process of addition can also be done in the downward direction i.e., addition of numbers column wise from top to bottom

EXAMPLE 1:

$$\begin{array}{r}
 4326 \\
 5\dot{6}9\dot{4} \\
 3708 \\
 \hline
 27\dot{8}4
 \end{array}$$

Step 1: $6 + 4 = 10$, 1 dot ; $0 + 8 = 8$; $8 + 4 = 12$; 1 dot and 2 answer under first column - total 2 dots.

Step 2: $2+2$ (2 dots) = 4; $4 + 9 = 13$: 1 dot and $3 + 0 = 3$; $3+8 = 11$; 1 dot and 1 answer under second column - total 2 dots.

Step 3: $3+2$ (2 dots) = 5; $5 + 6 = 11$: 1 dot and $1 + 7 = 8$; $8+7 = 15$; 1 dot and 5 under third column as answer - total 2 dots.

Step 4: $4 + 2$ (2 dots) = 6; $6 + 5 = 11$: 1 dot and $1 + 3 = 4$; $4 + 2 = 6$. - total 1 dot in the fourth 6 column as answer.

Step 5: 1 dot in the fourth column carried over to 5th column (No digits in it) as 1.

Thus answer is from Step 5 to Step 1; 16512

EXAMPLE 2:

$$\begin{array}{r} 3278 \\ 46\dot{1}9 \\ 308\dot{4} \\ \hline 1791 \end{array}$$

Steps

- (i) $8 + 9 = 17$; $7 + 4 = 11$; $1 + 1 = (2)$ (2dots)
- (ii) $7 + 2 = 9$; $9 + 1 = 10$; $0 + 8 = 8$, $8 + 9 = 17$, (7) (2dots)
- (iii) $2 + 2 = 4$; $4 + 6 = 10$; $0 + 0 = 0$; $0 + 7 = (7)$ (1 dot)
- (iv) $3 + 1 = 4$; $4 + 4 = 8$; $8 + 3 = 11$; $1 + 1 = (2)$ (1 dot)
- (v) 1

Thus answer is 12772.

Add the following numbers use 'Sudhikaran' wherever applicable.

1. 486

2. 5432

3. 968763

395

3691

476509

721

4808

+584376

+609

+6787

Check up whether 'Sudhkaran' is done correctly. If not write the correct process. In either case find the sums.

1. $\begin{array}{c} 4\dot{3} \\ 7\dot{8} \uparrow \\ +69 \end{array}$

2. $\begin{array}{c} 3\dot{7}\dot{9} \\ 8\dot{5}4 \\ 7\dot{6}\dot{7} \\ 426 \\ \hline \end{array}$

3. $\begin{array}{c} 78\dot{9}24 \downarrow \\ 2\dot{7}2\dot{7}2 \\ +726\dot{8}4 \end{array}$

Subtraction

The 'Sudha' Sutra is applicable where the larger digit is to be subtracted from the smaller digit. Let us go to the process through the examples.

Procedure:

- (i) If the digit to be subtracted is larger, a dot (sudha) is given to its left.
- (ii) The purak of this lower digit is added to the upper digit or purak-rekhanak of this lower digit is subtracted.

EXAMPLE 1:

$$34 - 18$$

$$\begin{array}{r} 34 \\ -\dot{1}8 \\ \hline \end{array}$$

Steps:

- (i) Since $8 > 4$, a dot is put on its left i.e. 1
- (ii) Purak of 8 i.e. 2 is added to the upper digit i.e. 4
 $2 + 4 = 6$. or Purak-rekhanak of 8 i.e. $\bar{2}$ is

Subtracted from i.e. $4 - \bar{2} = 6$.

Now at the tens place a dot (means 1) makes the '1' in the number into $1+1=2$. This has to be subtracted from above digit. i.e. $3 - 2 = 1$. Thus

$$\begin{array}{r} 34 \\ -18 \\ \hline 16 \end{array}$$

EXAMPLE 2:

$$\begin{array}{r} 63 \\ -37 \\ \hline \end{array}$$

Steps:

- (i) $7 > 3$. Hence a dot on left of 7 i.e., 3
- (ii) Purak of 7 i.e. 3 is added to upper digit 3 i.e. $3+3 = 6$.

This is unit place of the answer.

Thus answer is 26.

EXAMPLE 3:

$$\begin{array}{r} 3274 \\ -1892 \\ \hline \end{array}$$

Steps:

- (i) $2 < 4$. No sudha. $4-2 = 2$ first digit (from right to left)
- (ii) $9 > 7$ sudha required. Hence a dot on left of 9 i.e. $\dot{9}$
- (iii) purak of 9 i.e. 1, added to upper 7 gives $1 + 7 = 8$ second digit
- (iv) Now means $\dot{8} + 1 = 9$.

- (v) As $9 > 2$, once again the same process: dot on left of
i.e., $\dot{1}$
- (vi) purak of 9 i.e. 1, added to upper 2 gives $1 + 2 = 3$,
the third digit.
- (vii) Now $\dot{1}$ means $1 + 1 = 2$
- (viii) As $2 < 3$, we have $3 - 2 = 1$, the fourth digit
Thus answer is 1382

Vedic Check

E.g. (i) in addition : $437 + 624 + 586 + 162 = 1809$.

By beejank method, the Beejanks are

$$437 \longrightarrow 4 + 3 + 7 \longrightarrow 14 \quad 1 + 4 \longrightarrow 5$$

$$624 \longrightarrow 6 + 2 + 4 \longrightarrow 12 \longrightarrow 1 + 2 \longrightarrow 3$$

$$586 \longrightarrow 5 + 8 + 6 \longrightarrow 19 \longrightarrow 1 + 9 \longrightarrow$$

$$10 \longrightarrow 1 + 0 \longrightarrow 1$$

$$162 \longrightarrow 1 + 6 + 2 \longrightarrow 9$$

Now

$$437 + 624 + 586 + 162 \longrightarrow 5 + 3 + 1 + 9 \longrightarrow 18$$

$$\longrightarrow 1 + 8 \longrightarrow 9$$

$$\text{Beejank of } 1809 \longrightarrow 1 + 8 + 0 + 9 \longrightarrow 18 \quad 1 + 8$$

$$\longrightarrow 9 \text{ verified.}$$

E.g.(3) in subtraction :

$$3274 - 1892 = 1382$$

now beejanks

$$3274 \longrightarrow 3 + 2 + 7 + 4 \longrightarrow 3 + 4 \longrightarrow 7$$

$$1892 \longrightarrow 1 + 8 + 9 + 2 \longrightarrow 2$$

$$3292-1892 \longrightarrow 7-2 \longrightarrow 5$$

$$1382 \longrightarrow 1 + 3 + 8 + 2 \longrightarrow 5.$$

Hence verified.

Mixed addition and subtraction using Rekhanaks:

EXAMPLE 1 :

$$423 - 654 + 847 - 126 + 204.$$

In the conventional method we first add all the +ve terms

$$423 + 847 + 204 = 1474$$

Next we add all negative terms

$$- 654 - 126 = -780$$

At the end their difference is taken

$$1474 - 780 = 694$$

Thus in 3 steps we complete the problem

But in Vedic method using Rekhanak we write and directly find the answer.

$$4 \ 2 \ 3$$

$$\begin{array}{r} - \\ - \\ 6 \ 5 \ 4 \end{array}$$

$$8 \ 4 \ 7$$

$$\begin{array}{r} - \\ - \\ 1 \ 2 \ 6 \end{array}$$

$$2 \ 0 \ 4$$

$$\begin{array}{r} - \\ 7 \ 1 \ 4 \end{array}$$

This gives $(7 - 1) / (10 - 1) / 4 = 694.$

EXAMPLE (2):

$$6371 - 2647 + 8096 - 7381 + 1234$$

$$= 6371 + \overline{2647} + 8096 + \overline{7381} + 1234$$

$$= (6 + \overline{2} + 8 + \overline{7} + 1) / (3 + \overline{6} + 0 + \overline{3} + 2) /$$

$$(\overline{7} + 4 + \overline{9} + 8 + 3) / (\overline{1} + 7 + \overline{6} + 1 + 4)$$

$$= 6 / \overline{4} / 7 / 3$$

$$= (6 - 1) / (10 - 4) / 73$$

$$= 5673$$

Find the results in the following cases using Vedic methods.

1) $57 - 39$

2) $1286 - 968$

3) $384 - 127 + 696 - 549 + 150$

4) $7084 + 1232 - 6907 - 3852 + 4286$

*** Apply Vedic check for the above four problems and verify the results.**

Multiplication

We have already observed the application of Vedic sutras in multiplication. Let us recall them.

It enables us to have a comparative study of the applicability of these methods, to assess advantage of one method over the other method and so-on.

EXAMPLE 1:

- (i) By Ekadhikena purvena, since the number ends up in 5 we write the answer split up into two parts.

The right side part is 52 where as the left side part $19 \times (19+1)$ (Ekadhikena)

$$\text{Thus } 195^2 = 19 \times 20/5^2 = 380/25 = 38025$$

(ii) Find the square of 195.

The Conventional method :

$$\begin{array}{r}
 1952 = \quad 195 \\
 \quad \times 195 \\
 \hline
 \quad \quad 975 \\
 1755 \\
 195 \\
 \hline
 38025
 \end{array}$$

(iii) By Nikhilam Navatascaramam Dasatah; as the number is far from base 100, we combine the sutra with the upa-sutra 'anurupyena' and proceed by taking working base 200.

a) Working Base = 200 = 2 × 100.

Now 1952 = 195 × 195

$$\begin{array}{r}
 195 \qquad \qquad -5 \\
 195 \qquad \qquad -5 \\
 \hline
 (195-5) \text{ or } \quad -5 \times -5 \\
 \times 190 \qquad \quad = 25 \\
 \hline
 380 \qquad \quad / \quad 025 = 38025
 \end{array}$$

(iv) By the sutras "yavadunam tavadunikritya vargamca yojayet" and "anurupyena"

195², base 200 treated as 2 × 100 deficit is 5.

$$195^2 = \frac{(195-5)}{5^2}$$

$$= \frac{190 \times 2}{25}$$

operating on 100

$$= 38025$$

(v) By 'antyayor dasakepi' and 'Ekadhikena' sutras

Since in 195×195 , $5 + 5 = 10$ gives

$$195^2 = 19 \times 20 / 5 \times 5 = 380 / 25 = 38025.$$

EXAMPLE 2 :

$$98 \times 92$$

(i) 'Nikhilam' sutra

$$98 - 2$$

$$\times 92 - 8$$

$$90 / 16 = 9016$$

(ii) urdhava-tiryak sutra

$$98$$

$$\times 92$$

$$106$$

$$891$$

$$9016$$

(iii) by vinculum method

$$98 = 100 - 2 = 10\bar{2}$$

$$92 = 100 - 8 = 10\bar{8}$$

now

$$10\bar{2}$$

$$10\bar{8}$$

$$10006$$

$$\bar{1}1$$

$$11\bar{0}16 = 9016$$

(iv) 'Antyayordasakepi' and 'Ekadhikena' sutras.

98×92 Last digit sum = $8+2 = 10$ remaining digit(s) = 9 same sutras work.

$$\therefore 98 \times 92 = 9 \times (9 + 1) / 8 \times 2 = 90/16 = 9016.$$

EXAMPLE 3:

$$493 \times 497.$$

(1) 'Nikhilam' Method and 'Anurupyena':

(a) Working base is 500, treated as 5×100

$$\begin{array}{r} 493 \quad -7 \\ 497 \quad -3 \\ \hline 490 \quad / \quad 21 \quad \text{since base is } 5 \times 10 \\ \times 5 \\ \hline 2450 \quad / \quad 21 = 245021 \end{array}$$

(b) Working base is 500, treated as $1000 / 2$

$$493 \quad -7$$

$$497 \quad -3$$

$$2) \quad 490 / 021$$

$$245 / 021 = 245021$$

(2) Since end digits sum is $3+7 = 10$ and remaining part 49 is same in both the numbers, 'antyayordasakepi' is applicable. Further Ekadhikena Sutra is also applicable.

Thus,

$$\begin{aligned} 493 \times 497 &= 49 \times 50 / 3 \times 7 \\ &= 2450 / 21 \\ &= 245021 \end{aligned}$$

(3) With the use of vinculum.

$$493 = 500 - 07 = 50\bar{7}$$

$$497 = 500 - 03 = 50\bar{3}$$

Now 497×497 can be taken as $50\bar{7} \times 50\bar{3}$

$$\begin{array}{r} 50\bar{7} \\ \times 50\bar{3} \\ \hline 50001 \\ 252 \\ \hline 25\bar{5}021 = 245021 \end{array}$$

EXAMPLE 4:

$$99 \times 99$$

(1) Now by urdhva - tiryak sutra.

$$\begin{array}{r} 99 \\ \times 99 \\ \hline 8121 \\ 168 \\ \hline 9801 \end{array}$$

(2) By vinculum method

$$99 = 100 - 1 = 10\bar{1}$$

Now 99×99 is

$$\begin{array}{r} 10\bar{1} \\ \times 10\bar{1} \\ \hline 10\bar{2}01 = 9801 \end{array}$$

(3) By Nikhilam method

$$\begin{array}{r} 99 \quad -1 \\ 99 \quad -1 \\ \hline \end{array}$$

$$98 / 01 = 9801.$$

(4) 'Yadunam' sutra : $992 \text{ Base} = 100$

Deficiency is 1 : It indicates $992 = (99 - 1) / 1^2$

$$= 98 / 01 = 9801.$$

In the above examples we have observed how in more than one way problems can be solved and also the variety. You can have your own choice for doing multiplication. Not only that which method suits well for easier and quicker calculations. Thus the element of choice, divergent thinking, insight into properties and patterns in numbers, natural way of developing an idea, resourcefulness play major role in Vedic Mathematics methods.

Division

In the conventional procedure for division, the process is of the following form.

Quotient

Divisor) Dividend

or Divisor) Dividend (Quotient

Remainder

Remainder

But in the Vedic process, the format is:

Divisor) Dividend

Quotient / Remainder

The conventional method is always the same irrespective of the divisor. But Vedic methods are different depending on the nature of the divisor.

EXAMPLE 1:

Consider the division $1235 \div 89$.

(i) Conventional method:

89) 1235 (13

89

345

267

Thus $Q = 13$ and $R = 78$.

78

(ii) *Nikhilam method*: This method is useful when the divisor is nearer and less than the base. Since for 89, the base is 100 we can apply the method. Let us recall the *nikhilam* division already dealt.

Step (i): Write the dividend and divisor as in the conventional method. Obtain the modified divisor (M.D.) applying the *Nikhilam* formula. Write M.D. just below the actual divisor.

Thus for the divisor 89, the M.D. obtained by using *Nikhilam* is 11 in the last from 10 and the rest from 9. Now Step 1 gives

89) 1235

11

Step (ii): Bifurcate the dividend by a slash so that R.H.S of dividend contains the number of digits equal to that of M.D. Here M.D. contains 2 digits hence

89) 12 / 35

11

Step (iii): Multiply the M.D. with first column digit of the dividend. Here it is 1. i.e. $11 \times 1 = 11$. Write this product place wise under the 2nd and 3rd columns of the dividend.

$$89) 12 / 35$$

$$\begin{array}{r} \text{---} \\ 11 \quad 1 \quad 1 \end{array}$$

Step (iv): Add the digits in the 2nd column and multiply the M.D. with that result i.e. $2+1=3$ and $11 \times 3 = 33$. Write the digits of this result column wise as shown below, under 3rd and 4th columns. i.e.

$$89) 12 / 35$$

$$\begin{array}{r} \text{---} \\ 11 \quad 1 \quad 1 \\ \quad \quad 33 \end{array}$$

$$13 / 78$$

Now the division process is complete, giving $Q = 13$ and $R = 78$.

EXAMPLE 2:

Find Q and R for $121134 \div 8988$.

Steps (1+2):

$$8988) 12 / 1134$$

$$\begin{array}{r} \text{---} \\ 1012 \end{array}$$

Step (3):

$$8988) 12 / 1134$$

$$\begin{array}{r} \text{---} \\ 1012 \quad 1 \quad 012 \end{array}$$

Step(4):

$$8988) 12 / 1134$$

$$\begin{array}{r} \text{---} \\ 1012 \quad 1 \quad 012 \quad [\quad 2 + 1 = .3 \text{ and } 3 \times 1012 = 3036 \quad] \end{array}$$

$$3036$$

Now final Step

8988) 12 / 1134

1012 1 012

3036 (Column wise addition)

13 / 4290

Thus 121134, 8988 gives $Q = 13$ and $R = 4290$.

- (iii) Paravartya method: Recall that this method is suitable when the divisor is nearer but more than the base.

EXAMPLE 3:

$32894 \div 1028$.

The divisor has 4 digits. So the last 3 digits of the dividend are set apart for the remainder and the procedure follows.

1	0	2	8	32	8	9	4
	0	-2	-8	0	-6	-24	
					0	-4	16

32 2 -19 -12

- (1) $3 \times (0, -2, -8) = 0, -6, -24$
- (2) $2 - 0 = 2$ and $2 \times (0, -2, -8) = 0, -4, -16$
- (3) $8 - 6 + 0 = 2$
 $9 - 24 - 4 = -19$
 $4 - 16 = -12$

Now the remainder contains -19, -12 i.e. negative quantities. Observe that 32 is quotient. Take 1 over from the quotient column i.e. $1 \times 1028 = 1028$ over to the right side and proceed thus: $32 - 1 = 31$ becomes the Q and $R = 1028 + 200 - 190 - 12 = 1028 - 2 = 1026$.

Thus $3289 \div 1028$ gives $Q = 31$ and $R = 1026$.

EXAMPLE 4:

$$43852 \div 54.$$

Step 1:

Put down the first digit (5) of the divisor (54) in the divisor column as operator and the other digit (4) as flag digit. Separate the dividend into two parts where the right part has one digit. This is because the flag digit is single digit. The representation is as follows.

$$4 : 4 \ 3 \ 8 \ 5 : 2$$

5

Step 2:

- (i) Divide 43 by the operator 5. Now $Q=8$ and $R=3$. Write this $Q=8$ as the 1st Quotient - digit and prefix $R=3$, before the next digit i.e. 8 of the dividend, as shown below. Now 38 becomes the gross-dividend (G.D.) for the next step.

$$4 : 4 \ 3 \ 8 \ 5 : 2$$

$$5 : 3$$

$$: 8$$

- (ii) Subtract the product of flag digit (4) and first quotient digit (8) from the G.D. (38) i.e. $38 - (4 \times 8) = 38 - 32 = 6$. This is the net - dividend (N.D) for the next step.

Step 3:

Now N.D Operator gives Q and R as follows. $6 \div 5$, $Q=1$, $R=1$. So $Q=1$, the second quotient-digit and $R=1$, the prefix for the next digit (5) of the dividend.

$$4 : 4 \ 3 \ 8 \ 5 : 2$$

$$5 : 3 \ 1$$

$$: 8 \ 1$$

Step 4:

Now G.D = 15; product of flag-digit (4) and 2nd quotient-digit (1) is $4 \times 1 = 4$ Hence N.D = $15 - 4 = 11$ divide N.D by 5 to get $11 \div 5$, Q = 2, R = 1. The representation is

$$\begin{array}{r}
 4 : 4 \ 3 \ 8 \ 5 : 2 \\
 5 \quad : \quad 3 \ 1 : 1 \\
 \hline
 \quad : 8 \ 1 \ 2 :
 \end{array}$$

Step 5:

Now the R.H.S part has to be considered. The final remainder is obtained by subtracting the product of flag-digit (4) and third quotient digit (2) from 12 i.e., 12:

Final remainder = $12 - (4 \times 2) = 12 - 8 = 4$. Thus the division ends into

$$\begin{array}{r}
 4 : 4 \ 3 \ 8 \ 5 : 2 \\
 5 \quad : \quad 3 \ 1 : 1 \\
 \hline
 \quad : 8 \ 1 \ 2 : 4
 \end{array}$$

Thus $43852 \div 54$ gives Q = 812 and R = 4.

Consider the algebraic proof for the above problem. The divisor 54 can be represented by $5x + 4$, where $x = 10$

The dividend 43852 can be written algebraically as $43x^3 + 8x^2 + 5x + 2$

Since $x^3 = 10^3 = 1000$, $x^2 = 10^2 = 100$.

Now the division is as follows.

$$5x + 4) 43x^3 + 8x^2 + 5x + 2 \quad (8x^2 + x + 2)$$

$$43x^3 + 32x^2$$

$$3x^3 - 24x^2$$

$$= 6x^2 + 5x$$

$$(\because 3x^3 = 3 \cdot x \cdot x^2)$$

$$5x^2 + 4x$$

$$= 3 \cdot 10x^2 = 30x^2$$

$$x^2 + x$$

$$= 11x + 2 \quad (\because x^2 = x \cdot x = 10x)$$

$$\underline{10x + 8}$$

$$\quad \quad \quad x - 6$$

$$= 10 - 6 = 4.$$

Observe the Following Steps

1. $43x^3 \div 5x$ gives first quotient term $8x^2$, remainder $= 3x^3 - 24x^2$ which really mean $30x^2 + 8x^2 - 32x^2 = 6x^2$.

Thus in step 2 of the problem $43852 \div 54$, we get $Q = 8$ and $N.D = 6$.

2. $6x^2 \div 5x$ gives second quotient term x , remainder $= x^2 + x$ which really mean $10x + x = 11x$.

Thus in step 3 & Step 4, we get $Q = 1$ and $N.D = 11$.

3. $11x \div 5x$ gives third quotient term 2, remainder $= x - 6$, which really mean the final remainder $10 - 6 = 4$.

EXAMPLE 5:

Divide $237963 \div 524$

Step 1:

We take the divisor 524 as 5, the operator and 24, the flag-digit and proceed as in the above example. We now separate the dividend into two parts where the RHS part contains two digits for Remainder.

Thus,

$$24 : 2 \ 3 \ 7 \ 9 : 63$$

5

Step 2:

- (i) $23 \div 5$ gives $Q = 4$ and $R = 3$, $G.D = 37$.

- (ii) $N.D$ is obtained as

$$37 - \begin{pmatrix} 2 & 4 \\ 0 & 4 \end{pmatrix} \quad (\text{Flat digits})$$

$$= 37 - (8 + 0) = 29.$$

Representation

$$\begin{array}{r}
 24 : 2 \ 3 \ 7 \ 9 : 63 \\
 \quad \quad 5 \quad \quad 3 \\
 \hline
 : 4
 \end{array}$$

Step 3:

- (i) N.D $\div$ Operator = $29 \div 5$ gives Q = 5, R = 4 and G.D = 49.
 (ii) N.D is obtained as

$$\begin{aligned}
 49 - \left(\begin{array}{c} 2 \ 4 \\ \swarrow \searrow \\ 4 \ 5 \end{array} \right) & \text{ (Flat digits)} \\
 &= 49 - (10 + 16) \\
 &= 49 - 26 \\
 &= 23.
 \end{aligned}$$

i.e.,

$$\begin{array}{r}
 24 : 2 \ 3 \ 7 \ 9 : 63 \\
 \quad \quad 5 \quad : \quad 3 \ 4 : \\
 \hline
 : 4 \ 5 :
 \end{array}$$

Step 4:

- (i) N.D $\div$ Operator = $23 \div 5$ gives Q = 4, R = 3 and G.D = 363.

Note that we have reached the remainder part thus 363 is total sub-remainder.

$$\begin{array}{r}
 24 : 2 \ 3 \ 7 \ 9 : 63 \\
 \quad \quad 5 \quad : \quad 3 \quad 4 : 3 \\
 \hline
 : 4 \ 5 \ 4 :
 \end{array}$$

Step 5:

We find the final remainder as follows. Subtract the cross-product of the two, falg-digits and two last quotient-

digits and then vertical product of last flag-digit with last quotient-digit from the total sub-remainder.

i.e.,

$$\begin{aligned}
 363 - \left(\begin{array}{c} 2 \quad 4 \\ \swarrow \quad \searrow \\ 4 \quad 5 \end{array} \right) / \begin{array}{c} 4 \\ 1 \\ 4 \end{array} \\
 &= \frac{363 - (8 + 20)}{16} \\
 &= \frac{363 - 28}{16} \\
 &= 363 - 296 = 67.
 \end{aligned}$$

Note that 2, 4 are two flag digits: 5, 4 are two last quotient digits:

$$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

represents the last flag - digit and last quotient digit.

Thus the division $237963 \div 524$ gives $Q = 454$ and $R = 67$.

Thus the Vedic process of division which is also called as Straight division is a simple application of urdhva-tiryak together with dhvajanka. This process has many uses along with the one-line presentation of the answer.

Miscellaneous Items

Straight Squaring

We have already noticed methods useful to find out squares of numbers. But the methods are useful under some situations and conditions only. Now we go to a more general formula.

The sutra Dwandwa-yoga (Duplex combination process) is used in two different meanings. They are:

- i) by squaring
- ii) by cross-multiplying.

We use both the meanings of Dwandwa-yoga in the context of finding squares of numbers as follows:

We denote the Duplex of a number by the symbol D.

We define for a single digit 'a', $D = a^2$. and for a two digit number of the form 'ab', $D = 2(a \times b)$.

If it is a 3 digit number like 'abc', $D = 2(a \times c) + b^2$.

For a 4 digit number 'abcd', $D = 2(a \times d) + 2(b \times c)$ and so on. i.e. if the digit is single central digit, D represents 'square': and for the case of an even number of digits equidistant from the two ends D represent the double of the cross- product.

Consider the examples:

Number Duplex D

$$3 \quad 3^2 = 9$$

$$6 \quad 6^2 = 36$$

$$23 \quad 2(2 \times 3) = 12$$

$$64 \quad 2(6 \times 4) = 48$$

$$128 \quad 2(1 \times 8) + 2^2 = 16 + 4 = 20$$

$$305 \quad 2(3 \times 5) + 0^2 = 30 + 0 = 30$$

$$4231 \quad 2(4 \times 1) + 2(2 \times 3) = 8 + 12 = 20$$

$$7346 \quad 2(7 \times 6) + 2(3 \times 4) = 84 + 24 = 108$$

Further observe that for a n- digit number, the square of the number contains $2n$ or $2n-1$ digits.

Thus in this process, we take extra dots to the left one less than the number of digits in the given numbers.

EXAMPLES 1:

62^2 Since number of digits = 2, we take one extra dot to the left. Thus

$$\begin{array}{r} .62 \text{ for } 2, D = 22 = 4 \\ \hline \end{array}$$

$$644 \text{ for } 62, D = 2 \times 6 \times 2 = 24$$

$$\begin{array}{r} 32 \text{ for } 62, D = 2(0 \times 2) + 62 \\ \hline = 36 \end{array}$$

$$\hline 3844$$

$$\therefore 62^2 = 3844.$$

EXAMPLES 2:

234^2 Number of digits = 3. extradots = 2 Thus

$$\begin{array}{r} ..234 \text{ for } 4, D = 4^2 = 16 \\ \hline \end{array}$$

$$42546 \text{ for } 34, D = 2 \times 3 \times 4 = 24$$

$$\begin{array}{r} 1221 \text{ for } 234, D = 2 \times 2 \times 4 + 32 = 25 \\ \hline \end{array}$$

$$54756 \text{ for } .234, D = 2.0.4 + 2.2.3 = 12$$

$$\text{for } ..234, D = 2.0.4 + 2.0.3 + 22 = 4$$

EXAMPLES 3:

1426^2 . Number of digits = 4, extra dots = 3

i.e

$$\begin{array}{r} ...1426 \text{ } 6, D = 36 \\ \hline \end{array}$$

$$1808246 \text{ } 26, D = 2.2.6 = 24$$

$$\begin{array}{r} 22523 \text{ } 426, D = 2.4.6 + 22 = 52 \\ \hline \end{array}$$

$$2033476 \text{ } 1426, D = 2.1.6 + 2.4.2 = 28$$

$$.1426, D = 2.0.6 + 2.1.2 + 42 = 20$$

$$..1426, D = 2.0.6 + 2.0.2 + 2.1.4 = 8$$

$$...1426, D = 12 = 1$$

Thus $1426^2 = 2033476$.

With a little bit of practice the results can be obtained mentally as a single line answer.

ALGEBRAIC PROOF:

Consider the first example 62^2

$$\begin{aligned}\text{Now } 62^2 &= (6 \times 10 + 2)^2 = (10a + b)^2 \text{ where } a = 6, b = 2 \\ &= 100a^2 + 2.10a.b + b^2 \\ &= a^2 (100) + 2ab (10) + b^2\end{aligned}$$

i.e. b^2 in the unit place, $2ab$ in the 10th place and a^2 in the 100th place i.e. $2^2 = 4$ in units place, $2.6.2 = 24$ in the 10th place (4 in the 10th place and with carried over to 100th place). $6^2=36$ in the 100th place and with carried over 2 the 100th place becomes $36+2=38$.

Thus the answer 3844.

Find the squares of the numbers 54, 123, 2051, 3146.

Applying the Vedic sutra Dwanda yoga.

CUBING

Take a two digit number say 14.

- (i) Find the ratio of the two digits i.e. 1:4
- (ii) Now write the cube of the first digit of the number i.e. 13
- (iii) Now write numbers in a row of 4 terms in such a way that the first one is the cube of the first digit and remaining three are obtained in a geometric progression with common ratio as the ratio of the original two digits (i.e. 1:4) i.e. the row is

$$1 \quad 4 \quad 16 \quad 64.$$
- (iv) Write twice the values of 2nd and 3rd terms under the terms respectively in second row.

i.e.,

$$1 \ 4 \ 16 \ 64$$

$$8 \ 32 \ (2 \times 4 = 8, 2 \times 16 = 32)$$

- (v) Add the numbers column wise and follow carry over process.

$$1 \ 4 \ 16 \ 64 \quad \text{Since } 16 + 32 + 6 \text{ (carryover)} = 54$$

$$8 \ 32 \ 4 \quad \text{written and } 5 \text{ (carryover)} + 4 + 8 = 17$$

$$2 \ 7 \ 4 \ 4 \quad 7 \text{ written and } 1 \text{ (carryover)} + 1 = 2.$$

This 2744 is nothing but the cube of the number 14

ALGEBRAIC PROOF:

Let a and b be two digits.

Consider the row $a^3 \ a^2b \ ab^2 \ b^3$

the first is a^3 and the numbers are in the ratio a:b

$$\text{since } a^3 : a^2b = a^2b : b^3 = a : b$$

Now twice of a^2b , ab^2 are $2a^2b$, $2ab^2$

$$a^3 + a^2b + ab^2 + b^3$$

$$2a^2b + 2ab^2$$

$$a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3.$$

Thus cubes of two digit numbers can be obtained very easily by using the vedic sutra 'anurupyena'. Now cubing can be done by using the vedic sutra 'Yavadunam'.

EXAMPLE 3:

Consider 1063.

- (i) The base is 100 and excess is 6. In this context we double the excess and then add.

$$\text{i.e. } 106 + 12 = 118. (\because 2 \times 6 = 12)$$

This becomes the left - hand - most portion of the cube.

$$\text{i.e. } 106^3 = 118 / - - -$$

- (ii) Multiply the new excess by the initial excess

$$\text{i.e. } 18 \times 6 = 108 \text{ (excess of 118 is 18)}$$

Now this forms the middle portion of the product of course 1 is carried over, 08 in the middle.

$$\text{i.e. } 106^3 = 118 / 08 / - - - -$$

1

- (iii) The last portion of the product is cube of the initial excess.

$$\text{i.e. } 6^3 = 216.$$

16 in the last portion and 2 carried over.

$$\text{i.e. } 106^3 = 118 / 081 / 16 = 1191016$$

1 2

EXAMPLE 4:

Find 1002^3 .

- (i) Base = 1000. Excess = 2. Left-hand-most portion of the cube becomes $1002 + (2 \times 2) = 1006$.
- (ii) New excess $\times$ initial excess = $6 \times 2 = 12$.
- Thus 012 forms the middle portion of the cube.
- (iii) Cube of initial excess = $2^3 = 8$.

So the last portion is 008.

$$\text{Thus } 1002^3 = 1006 / 012 / 008 = 1006012008.$$

EXAMPLE 5:

Find 94^3 .

- (i) Base = 100, deficit = -6. Left-hand-most portion of the cube becomes $94 + (2 \times -6) = 94 - 12 = 82$.
- (ii) New deficit $\times$ initial deficit
- $$= -(100 - 82) \times (-6) = -18 \times -6 = 108$$

Thus middle portion of the cube = 08 and 1 is carried over.

(iii) Cube of initial deficit = $(-6)^3 = -216$

Now $943 = 82 / 08 / \overline{16} = 83 / 06 / \overline{16}$

1 $\overline{2}$

= $83 / 05 / (100 - 16)$

= 830584.

Find the cubes of the following numbers using Vedic sutras.

103, 112, 91, 89, 998, 9992, 1014.

Equation of Straight line passing through two given points:

To find the equation of straight line passing through the points (x_1, y_1) and (x_2, y_2) , we generally consider one of the following methods.

1. General equation $y = mx + c$.

It is passing through (x_1, y_1) then $y_1 = mx_1 + c$.

It is passing through (x_2, y_2) also, then $y_2 = mx_2 + c$.

Solving these two simultaneous equations, we get 'm' and 'c' and so the equation.

2. The formula:

$$y - y_1 \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \text{ and substitution.}$$

Some sequence of steps gives the equation. But the paravartya sutra enables us to arrive at the conclusion in a more easy way and convenient to work mentally.

EXAMPLE 1 :

Find the equation of the line passing through the points (9,7) and (5,2).

Step 1: Put the difference of the y - coordinates as the x- coefficient and vice - versa.

$$\text{i.e. x coefficient} = 7 - 2 = 5$$

$$\text{y coefficient} = 9 - 5 = 4.$$

Thus L.H.S of equation is $5x - 4y$.

Step 2: The constant term (R.H.S) is obtained by substituting the coordinates of either of the given points in

L.H.S (obtained through step-1)

i.e. R.H.S of the equation is

$$5(9) - 4(7) = 45 - 28 = 17$$

$$\text{or } 5(5) - 4(2) = 25 - 8 = 17.$$

Thus the equation is $5x - 4y = 17$.

EXAMPLE 2:

Find the equation of the line passing through (2, -3) and (4,-7).

$$\text{Step 1 : } x[-3-(-7)] - y[2-4] = 4x + 2y.$$

$$\text{Step 2 : } 4(2) + 2(-3) = 8 - 6 = 2.$$

$$\text{Step 3 : Equation is } 4x + 2y = 2 \text{ or } 2x + y = 1.$$

EXAMPLE 3 :

Equation of the line passing through the points (7,9) and (3,-7).

$$\text{Step 1 : } x[9 - (-7)] - y(7 - 3) = 16x - 4y.$$

$$\text{Step 2 : } 16(7) - 4(9) = 112 - 36 = 76$$

$$\text{Step 3 : } 16x - 4y = 76 \text{ or } 4x - y = 19$$

Find the equation of the line passing through the points using Vedic methods.

1. (1, 2), (4,-3)
2. (5,-2), (5,-4)

3. $(-5, -7), (13, 2)$

4. $(a, 0), (0, b)$

Conclusion

After going through the content presented in this book, you may, perhaps, have noted a number of applications of methods of Vedic Mathematics. We are aware that this attempt is only to make you familiar with a few special methods.

The methods discussed, and organization of the content here are intended for any reader with some basic mathematical background. That is why the serious mathematical issues, higher level mathematical problems are not taken up in this volume, even though many aspects like four fundamental operations, squaring, cubing, linear equations, simultaneous equations, factorization, H.C.F, recurring decimals, etc. are dealt with.

Many more concepts and aspects are omitted unavoidably, keeping in view the scope and limitations of the present volume.

Thus the present volume serves as only an 'introduction'. More has to be presented to cover all the issues in Swamiji's 'Vedic Mathematics'. Still more steps are needed to touch the latest developments in Vedic Mathematics.

As a result, serious and sincere work by scholars and research workers continues in this field both in our country and abroad. Sri Sathya Sai Veda Pratisthan intends to bring about more volumes covering the aspects now left over, and also elaborating the content of Vedic Mathematics.

The present volume, even though introductory, has touched almost all the Sutras and sub-Sutras as mentioned in Swamiji's 'Vedic Mathematics'. Further it has given rationale and proof for the methods. As there is a general opinion that the 'so called Vedic Mathematics is only rude,

rote, non mathematical and none other than some sort of tricks', the logic, proof and Mathematics behind the 'the so called tricks' has been explained.

An impartial reader can easily experience the beauty, charm and resourcefulness in Vedic Mathematics systems. We feel that the reader can enjoy the diversity and simplicity in Vedic Mathematics while applying the methods against the conventional textbook methods. The reader can also compare and contrast both the methods.

The Vedic Methods enable the practitioner improve mental abilities to solve difficult problems with high speed and accuracy.

History of Mathematics in India

In all early civilizations, the first expression of mathematical understanding appears in the form of counting systems. Numbers in very early societies were typically represented by groups of lines, though later different numbers came to be assigned specific numeral names and symbols (as in India) or were designated by alphabetic letters (such as in Rome). Although today, we take our decimal system for granted, not all ancient civilizations based their numbers on a ten-base system. In ancient Babylon, a sexagesimal (base 60) system was in use.

The Decimal System in Harappa

In India a decimal system was already in place during the Harappan period, as indicated by an analysis of Harappan weights and measures. Weights corresponding to ratios of 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50, 100, 200, and 500 have been identified, as have scales with decimal divisions. A particularly notable characteristic of Harappan weights and measures is their remarkable accuracy. A bronze rod marked in units of 0.367 inches points to the degree of precision demanded in those times. Such scales were particularly important in ensuring proper implementation of town planning rules that required roads of fixed widths to run at

right angles to each other, for drains to be constructed of precise measurements, and for homes to be constructed according to specified guidelines. The existence of a graded system of accurately marked weights points to the development of trade and commerce in Harappan society.

Mathematical Activity in the Vedic Period

In the Vedic period, records of mathematical activity are mostly to be found in Vedic texts associated with ritual activities. However, as in many other early agricultural civilizations, the study of arithmetic and geometry was also impelled by secular considerations. Thus, to some extent early mathematical developments in India mirrored the developments in Egypt, Babylon and China.

The system of land grants and agricultural tax assessments required accurate measurement of cultivated areas. As land was redistributed or consolidated, problems of mensuration came up that required solutions. In order to ensure that all cultivators had equivalent amounts of irrigated and non-irrigated lands and tracts of equivalent fertility — individual farmers in a village often had their holdings broken up in several parcels to ensure fairness. Since plots could not all be of the same shape — local administrators were required to convert rectangular plots or triangular plots to squares of equivalent sizes and so on. Tax assessments were based on fixed proportions of annual or seasonal crop incomes, but could be adjusted upwards or downwards based on a variety of factors. This meant that an understanding of geometry and arithmetic was virtually essential for revenue administrators. Mathematics was thus brought into the service of both the secular and the ritual domains.

Arithmetic operations (Ganit) such as addition, subtraction, multiplication, fractions, squares, cubes and roots are enumerated in the Narad Vishnu Purana attributed

to Ved Vyas (pre-1000 BC). Examples of geometric knowledge (rekha-ganit) are to be found in the Sulva-Sutras of Baudhayana (800 BC) and Apasthamba (600 BC) which describe techniques for the construction of ritual altars in use during the Vedic era. It is likely that these texts tapped geometric knowledge that may have been acquired much earlier, possibly in the Hārappan period. Baudhayana's Sutra displays an understanding of basic geometric shapes and techniques of converting one geometric shape (such as a rectangle) to another of equivalent (or multiple, or fractional) area (such as a square). While some of the formulations are approximations, others are accurate and reveal a certain degree of practical ingenuity as well as some theoretical understanding of basic geometric principles. Modern methods of multiplication and addition probably emerged from the techniques described in the Sulva-Sutras.

Pythagoras — the Greek mathematician and philosopher who lived in the 6th CBC, was familiar with the Upanishads and learnt his basic geometry from the Sulva Sutras. An early statement of what is commonly known as the Pythagoras theorem is to be found in Baudhayana's Sutra: The chord which is stretched across the diagonal of a square produces an area of double the size. A similar observation pertaining to oblongs is also noted. His Sutra also contains geometric solutions of a linear equation in a single unknown. Examples of quadratic equations also appear. Apasthamba's sutra (an expansion of Baudhayana's with several original contributions) provides a value for the square root of 2 that is accurate to the fifth decimal place. Apasthamba also looked at the problems of squaring a circle, dividing a segment into seven equal parts, and a solution to the general linear equation. Jain texts from the 6th C BC such as the Surya Pragyapti describe ellipses.

Modern-day commentators are divided on how some of the results were generated. Some believe that these results came about through hit and trial — as rules of thumb, or

as generalizations of observed examples. Others believe that once the scientific method came to be formalized in the Nyaya-Sutras — proofs for such results must have been provided, but these have either been lost or destroyed, or else were transmitted orally through the Gurukul system, and only the final results were tabulated in the texts. In any case, the study of Ganit i.e. mathematics was given considerable importance in the Vedic period. The Vedang Jyotish (1000 BC) includes the statement: "Just as the feathers of a peacock and the jewel-stone of a snake are placed at the highest point of the body (at the forehead), similarly, the position of Ganit is the highest amongst all branches of the Vedas and the Shastras."

Many centuries later, Jain mathematician from Mysore, Mahaviracharya further emphasized the importance of mathematics: "Whatever object exists in this moving and non-moving world, cannot be understood without the base of Ganit (i.e. mathematics)".

Panini and Formal Scientific Notation

A particularly important development in the history of Indian science that was to have a profound impact on all mathematical treatises that followed was the pioneering work by Panini (6th C BC) in the field of Sanskrit grammar and linguistics. Besides expounding a comprehensive and scientific theory of phonetics, phonology and morphology, Panini provided formal production rules and definitions describing Sanskrit grammar in his treatise called *Asthadhyayi*. Basic elements such as vowels and consonants, parts of speech such as nouns and verbs were placed in classes. The construction of compound words and sentences was elaborated through ordered rules operating on underlying structures in a manner similar to formal language theory.

Today, Panini's constructions can also be seen as comparable to modern definitions of a mathematical function. G. G. Joseph, in *The crest of the peacock* argues that the

algebraic nature of Indian mathematics arises as a consequence of the structure of the Sanskrit language. Ingerman in his paper titled Panini-Backus form finds Panini's notation to be equivalent in its power to that of Backus — inventor of the Backus Normal Form used to describe the syntax of modern computer languages. Thus Panini's work provided an example of a scientific notational model that could have propelled later mathematicians to use abstract notations in characterizing algebraic equations and presenting algebraic theorems and results in a scientific format.

Philosophy and Mathematics

Philosophical doctrines also had a profound influence on the development of mathematical concepts and formulations. Like the Upanishadic world view, space and time were considered limitless in Jain cosmology. This led to a deep interest in very large numbers and definitions of infinite numbers. Infinite numbers were created through recursive formulae, as in the Anuyoga Dwara Sutra. Jain mathematicians recognized five different types of infinities: infinite in one direction, in two directions, in area, infinite everywhere and perpetually infinite. Permutations and combinations are listed in the Bhagvati Sutras (3rd C BC) and Sathananga Sutra (2nd C BC).

Jain set theory probably arose in parallel with the Syadvada system of Jain epistemology in which reality was described in terms of pairs of truth conditions and state changes. The Anuyoga Dwara Sutra demonstrates an understanding of the law of indices and uses it to develop the notion of logarithms. Terms like Ardh Aached, Trik Aached, and Chatur Aached are used to denote log base 2, log base 3 and log base 4 respectively. In Satkhandagama various sets are operated upon by logarithmic functions to base two, by squaring and extracting square roots, and by raising to finite or infinite powers. The operations are

repeated to produce new sets. In other works the relation of the number of combinations to the coefficients occurring in the binomial expansion is noted.

Since Jain epistemology allowed for a degree of indeterminacy in describing reality, it probably helped in grappling with indeterminate equations and finding numerical approximations to irrational numbers.

Buddhist literature also demonstrates an awareness of indeterminate and infinite numbers. Buddhist mathematics was classified either as *Garna* (Simple Mathematics) or *Sankhyān* (Higher Mathematics). Numbers were deemed to be of three types: *Sankheya* (countable), *Asankheya* (uncountable) and *Anant* (infinite).

Philosophical formulations concerning *Shunya* — i.e. emptiness or the void may have facilitated in the introduction of the concept of zero. While the zero (*bindu*) as an empty place holder in the place-value numeral system appears much earlier, algebraic definitions of the zero and its relationship to mathematical functions appear in the mathematical treatises of Brahmagupta in the 7th C AD. Although scholars are divided about how early the symbol for zero came to be used in numeric notation in India, (Ifrah arguing that the use of zero is already implied in Aryabhata) tangible evidence for the use of the zero begins to proliferate towards the end of the Gupta period. Between the 7th C and the 11th C, Indian numerals developed into their modern form, and along with the symbols denoting various mathematical functions (such as plus, minus, square root etc.) eventually became the foundation stones of modern mathematical notation.

The Indian Numeral System

Although the Chinese were also using a decimal based counting system, the Chinese lacked a formal notational system that had the abstraction and elegance of the Indian

notational system, and it was the Indian notational system that reached the Western world through the Arabs and has now been accepted as universal. Several factors contributed to this development whose significance is perhaps best stated by French mathematician, Laplace: "The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. It's simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions."

Brilliant as it was, this invention was no accident. In the Western world, the cumbersome roman numeral system posed as a major obstacle, and in China the pictorial script posed as a hindrance. But in India, almost everything was in place to favour such a development. There was already a long and established history in the use of decimal numbers, and philosophical and cosmological constructs encouraged a creative and expansive approach to number theory. Panini's studies in linguistic theory and formal language and the powerful role of symbolism and representational abstraction in art and architecture may have also provided an impetus, as might have the rationalist doctrines and the exacting epistemology of the Nyaya Sutras, and the innovative abstractions of the Syadavada and Buddhist schools of learning.

Influence of Trade and Commerce, Importance of Astronomy

The growth of trade and commerce, particularly lending and borrowing demanded an understanding of both simple and compound interest which probably stimulated the interest in arithmetic and geometric series. Brahmagupta's description of negative numbers as debts and positive numbers as fortunes points to a link between trade and

mathematical study. Knowledge of astronomy — particularly knowledge of the tides and the stars was of great import to trading communities who crossed oceans or deserts at night. This is borne out by numerous references in the Jataka tales and several other folk-tales. The young person who wished to embark on a commercial venture was inevitably required to first gain some grounding in astronomy. This led to a proliferation of teachers of astronomy, who in turn received training at universities such as at Kusumpura (Bihar) or Ujjain (Central India) or at smaller local colleges or Gurukuls. This also led to the exchange of texts on astronomy and mathematics amongst scholars and the transmission of knowledge from one part of India to another. Virtually every Indian state produced great mathematicians who wrote commentaries on the works of other mathematicians (who may have lived and worked in a different part of India many centuries earlier). Sanskrit served as the common medium of scientific communication.

The science of astronomy was also spurred by the need to have accurate calendars and a better understanding of climate and rainfall patterns for timely sowing and choice of crops. At the same time, religion and astrology also played a role in creating an interest in astronomy and a negative fallout of this irrational influence was the rejection of scientific theories that were far ahead of their time. One of the greatest scientists of the Gupta period — Aryabhata (born in 476 AD, Kusumpura, Bihar) provided a systematic treatment of the position of the planets in space. He correctly posited the axial rotation of the earth, and inferred correctly that the orbits of the planets were ellipses. He also correctly deduced that the moon and the planets shined by reflected sunlight and provided a valid explanation for the solar and lunar eclipses rejecting the superstitions and mythical belief systems surrounding the phenomenon.

Although Bhaskar I (born Saurashtra, 6th C, and follower of the Asmaka school of science, Nizamabad, Andhra)

recognized his genius and the tremendous value of his scientific contributions, some later astronomers continued to believe in a static earth and rejected his rational explanations of the eclipses. But in spite of such setbacks, Aryabhatta had a profound influence on the astronomers and mathematicians who followed him, particularly on those from the Asmaka school.

Mathematics played a vital role in Aryabhatta's revolutionary understanding of the solar system. His calculations on π , the circumference of the earth (62832 miles) and the length of the solar year (within about 13 minutes of the modern calculation) were remarkably close approximations. In making such calculations, Aryabhatta had to solve several mathematical problems that had not been addressed before, including problems in algebra (*beej-ganit*) and trigonometry (*trikonmiti*).

Bhaskar I continued where Aryabhatta left off, and discussed in further detail topics such as the longitudes of the planets; conjunctions of the planets with each other and with bright stars; risings and settings of the planets; and the lunar crescent. Again, these studies required still more advanced mathematics and Bhaskar I expanded on the trigonometric equations provided by Aryabhatta, and like Aryabhatta correctly assessed π to be an irrational number. Amongst his most important contributions was his formula for calculating the sine function which was 99% accurate. He also did pioneering work on indeterminate equations and considered for the first time quadrilaterals with all the four sides unequal and none of the opposite sides parallel.

Another important astronomer/mathematician was Varahamira (6th C, Ujjain) who compiled previously written texts on astronomy and made important additions to Aryabhatta's trigonometric formulas. His works on permutations and combinations complemented what had been previously achieved by Jain mathematicians and

provided a method of calculation of nCr that closely resembles the much more recent Pascal's Triangle. In the 7th century, Brahmagupta did important work in enumerating the basic principles of algebra. In addition to listing the algebraic properties of zero, he also listed the algebraic properties of negative numbers. His work on solutions to quadratic indeterminate equations anticipated the work of Euler and Lagrange.

Emergence of Calculus

In the course of developing a precise mapping of the lunar eclipse, Aryabhata was obliged to introduce the concept of infinitesimals — i.e. *tatkalika gati* to designate the infinitesimal, or near instantaneous motion of the moon, and express it in the form of a basic differential equation. Aryabhata's equations were elaborated on by Manjula (10th C) and Bhaskaracharya (12th C) who derived the differential of the sine function. Later mathematicians used their intuitive understanding of integration in deriving the areas of curved surfaces and the volumes enclosed by them.

Applied Mathematics, Solutions to Practical Problems

Developments also took place in applied mathematics such as in creation of trigonometric tables and measurement units. Yativrsabha's work *Tiloyapannatti* (6th C) gives various units for measuring distances and time and also describes the system of infinite time measures.

In the 9th C, Mahaviracharya (Mysore) wrote *Ganit Saar Sangraha* where he described the currently used method of calculating the Least Common Multiple (LCM) of given numbers. He also derived formulae to calculate the area of an ellipse and a quadrilateral inscribed within a circle (something that had also been looked at by Brahmagupta) The solution of indeterminate equations also drew

considerable interest in the 9th century, and several mathematicians contributed approximations and solutions to different types of indeterminate equations.

In the late 9th C, Sridhara (probably Bengal) provided mathematical formulae for a variety of practical problems involving ratios, barter, simple interest, mixtures, purchase and sale, rates of travel, wages, and filling of cisterns. Some of these examples involved fairly complicated solutions and his *Patiganita* is considered an advanced mathematical work. Sections of the book were also devoted to arithmetic and geometric progressions, including progressions with fractional numbers or terms, and formulas for the sum of certain finite series are provided. Mathematical investigation continued into the 10th C. Vijayanandi (of Benares, whose *Karanatilaka* was translated by Al-Beruni into Arabic) and Sripati of Maharashtra are amongst the prominent mathematicians of the century.

The leading light of 12th C Indian mathematics was Bhaskaracharya who came from a long-line of mathematicians and was head of the astronomical observatory at Ujjain. He left several important mathematical texts including the *Lilavati* and *Bijaganita* and the *Siddhanta Shiromani*, an astronomical text. He was the first to recognize that certain types of quadratic equations could have two solutions. His *Chakrawaat* method of solving indeterminate solutions preceded European solutions by several centuries, and in his *Siddhanta Shiromani* he postulated that the earth had a gravitational force, and broached the fields of infinitesimal calculation and integration.

In the second part of this treatise, there are several chapters relating to the study of the sphere and its properties and applications to geography, planetary mean motion, eccentric epicyclical model of the planets, first visibilities of the planets, the seasons, the lunar crescent etc. He also discussed astronomical instruments and spherical

trigonometry. Of particular interest are his trigonometric equations:

$$\sin(a + b) = \sin a \cos b + \cos a \sin b;$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b;$$

The Spread of Indian Mathematics

The study of mathematics appears to slow down after the onslaught of the Islamic invasions and the conversion of colleges and universities to madrassas. But this was also the time when Indian mathematical texts were increasingly being translated into Arabic and Persian. Although Arab scholars relied on a variety of sources including Babylonian, Syrian, Greek and some Chinese texts, Indian mathematical texts played a particularly important role.

Scholars such as Ibn Tariq and Al-Fazari (8th C, Baghdad), Al-Kindi (9th C, Basra), Al-Khwarizmi (9th C, Khiva), Al-Qayarawani (9th C, Maghreb, author of *Kitab fi al-hisab al-hindi*), Al-Uqlidisi (10th C, Damascus, author of *The book of Chapters in Indian Arithmetic*), Ibn-Sina (Avicenna), Ibn al-Samh (Granada, 11th C, Spain), Al-Nasawi (Khurasan, 11th C, Persia), Al-Beruni (11th C, born Khiva, died Afghanistan), Al-Razi (Teheran), and Ibn-Al-Saffar (11th C, Cordoba) were amongst the many who based their own scientific texts on translations of Indian treatises. Records of the Indian origin of many proofs, concepts and formulations were obscured in the later centuries, but the enormous contributions of Indian mathematics was generously acknowledged by several important Arabic and Persian scholars, especially in Spain. Abbasid scholar Al-Gaheth wrote: "India is the source of knowledge, thought and insight". Al-Maoudi (956 AD) who travelled in Western India also wrote about the greatness of Indian science. Said Al-Andalusi, an 11th C Spanish scholar and court historian was amongst the most enthusiastic in his praise of Indian civilization, and specially remarked on Indian achievements in the sciences

and in mathematics. Of course, eventually, Indian algebra and trigonometry reached Europe through a cycle of translations, travelling from the Arab world to Spain and Sicily, and eventually penetrating all of Europe. At the same time, Arabic and Persian translations of Greek and Egyptian scientific texts became more readily available in India.

The Kerala School

Although it appears that original work in mathematics ceased in much of Northern India after the Islamic conquests, Benaras survived as a centre for mathematical study, and an important school of mathematics blossomed in Kerala. Madhava (14th C, Kochi) made important mathematical discoveries that would not be identified by European mathematicians till at least two centuries later. His series expansion of the cos and sine functions anticipated Newton by almost three centuries. Historians of mathematics, Rajagopal, Rangachari and Joseph considered his contributions instrumental in taking mathematics to the next stage, that of modern classical analysis.

Nilkantha (15th C, Tirur, Kerala) extended and elaborated upon the results of Madhava while Jyesthadeva (16th C, Kerala) provided detailed proofs of the theorems and derivations of the rules contained in the works of Madhava and Nilkantha. It is also notable that Jyesthadeva's *Yuktibhasa* which contained commentaries on Nilkantha's *Tantrasamgraha* included elaborations on planetary theory later adopted by Tycho Brahe, and mathematics that anticipated work by later Europeans. Chitrabhanu (16th C, Kerala) gave integer solutions to twenty-one types of systems of two algebraic equations, using both algebraic and geometric methods in developing his results. Important discoveries by the Kerala mathematicians included the Newton-Gauss interpolation formula, the formula for the sum of an infinite series, and a series notation for π . Charles Whish (1835, published in the *Transactions of the Royal Asiatic Society*

of Great Britain and Ireland) was one of the first Westerners to recognize that the Kerala school had anticipated by almost 300 years many European developments in the field.

Yet, few modern compendiums on the history of mathematics have paid adequate attention to the often pioneering and revolutionary contributions of Indian mathematicians. But as this essay amply demonstrates, a significant body of mathematical works were produced in the Indian subcontinent.

The science of mathematics played a pivotal role not only in the industrial revolution but in the scientific developments that have occurred since. No other branch of science is complete without mathematics. Not only did India provide the financial capital for the industrial revolution India also provided vital elements of the scientific foundation without which humanity could not have entered this modern age of science and high technology.

Notes: Mathematics and Music: Pingala (3rd C AD), author of Chandasutra explored the relationship between combinatorics and musical theory anticipating Mersenne (1588-1648) author of a classic on musical theory.

Mathematics and Architecture: Interest in arithmetic and geometric series may have also been stimulated by (and influenced) Indian architectural designs - (as in temple shikaras, gopurams and corbelled temple ceilings). Of course, the relationship between geometry and architectural decoration was developed to it's greatest heights by Central Asian, Persian, Turkish, Arab and Indian architects in a variety of monuments commissioned by the Islamic rulers.

Transmission of the Indian Numeral System: Evidence for the transmission of the Indian Numeral System to the West is provided by Joseph (Crest of the Peacock):

Quotes Severus Sebokht (662) in a Syriac text describing the "subtle discoveries" of Indian astronomers as being "more

ingenious than those of the Greeks and the Babylonians" and "their valuable methods of computation which surpass description" and then goes on to mention the use of nine numerals.

Quotes from *Liber abaci* (Book of the Abacus) by Fibonacci (1170-1250): The nine Indian numerals are ...with these nine and with the sign 0 which in Arabic is sifr, any desired number can be written. (Fibonacci learnt about Indian numerals from his Arab teachers in North Africa)

Influence of the Kerala School: Joseph (Crest of the Peacock) suggests that Indian mathematical manuscripts may have been brought to Europe by Jesuit priests such as Matteo Ricci who spent two years in Kochi (Cochin) after being ordained in Goa in 1580. Kochi is only 70km from Thrissur (Trichur) which was then the largest repository of astronomical documents.

Whish and Hyne — two European mathematicians obtained their copies of works by the Kerala mathematicians from Thrissur, and it is not inconceivable that Jesuit monks may have also taken copies to Pisa (where Galileo, Cavalieri and Wallis spent time), or Padua (where James Gregory studied) or Paris (where Mersenne who was in touch with Fermat and Pascal, acted as an agent for the transmission of mathematical ideas).

Indic Mathematics

Math and Ethnocentrism

The study of mathematics in the West has long been characterized by a certain ethnocentric bias, a bias which most often manifests not in explicit racism, but in a tendency toward undermining or eliding the real contributions made by non-Western civilizations. The debt owed by the West to other civilizations, and to India in particular, go back to the earliest epoch of the "Western" scientific tradition, the age

of the classical Greeks, and continued up until the dawn of the modern era, the renaissance, when Europe was awakening from its dark ages. This awakening was in part made possible by the rediscovery of mathematics and other sciences and technologies through the medium of the Arabs, who transmitted to Europe both their own lost heritage as well as the advanced mathematical traditions formulated in India.

George Ghevarughese Joseph, in an important article entitled "Foundations of Eurocentrism in Mathematics," argued that "the standard treatment of the history of non-European mathematics is a product of historiographical bias (conscious or otherwise) in the selection and interpretation of facts, which, as a consequence, results in ignoring, devaluing or distorting contributions arising outside European mathematical traditions." (1987:14)

Due to the legacy of colonialism, the exploitation of which was ideologically justified through a doctrine of racial superiority, the contributions of non-European civilizations were often ignored, or, as Joseph argued, even distorted, in that they were often misattributed as European, i.e. Greek, contributions, and when their contributions were so great as to resist such treatment, they were typically devalued, considered inferior or irrelevant to Western mathematical traditions.

This tendency has not only led to the devaluation of non-Western mathematical traditions, but has distorted the history of Western mathematics as well. In so far as the contributions from non-Western civilizations are ignored, there is the problem of accounting for the development of mathematics purely within the Western cultural framework.

This has led, as Sabetai Unguru has argued, toward a tendency to read more advanced mathematical concepts into the relatively simplistic geometrical formulations of Greek

mathematicians such as Euclid, despite the fact that the Greeks lacked not only mathematic notation, but even the place-value system of enumeration, without which advanced mathematical calculation is impossible. Such ethnocentric revisionist history resulted in the attribution of more advanced algebraic concepts, which were actually introduced to Europe over a millennium later by the Arabs, to the Greeks.

And while the contributions of the Greeks to mathematics was quite significant, the tendency of some math historians to jump from the Greeks to renaissance Europe results not only in an ethnocentric history, but an inadequate history as well, one which fails to take into account the full history of the development of modern mathematics, which is by no means a purely European development.

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Dr. Gray goes on to list some of the most important developments in the history of mathematics that took place in India, summarizing the contributions of luminaries such as Aryabhatta, Brahmagupta, Mahavira, Bhaskara and Madhava. He concludes by asserting that "the role played by India in the development (of the scientific revolution in Europe) is no mere footnote, easily and inconsequentially swept under the rug of Eurocentric bias. To do so is to distort history, and to deny India one of its greatest contributions to world civilization." -Indic Mathematics

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Vedic Altars and the "Pythagorean Theorem"

A perfect example of this sort of misattribution involves the so-called Pythagorean theorem, the well-known theorem which was attributed to Pythagoras who lived around 500 BCE, but which was first proven in Greek sources in Euclid's *Geometry*, written centuries later. Despite the scarcity of evidence backing this attribution, it is not often questioned, perhaps due to the mantra-like frequency with which it is repeated. However, Seidenberg, in his 1978 article, shows that the thesis that Greece was the origin of geometric algebra was incorrect, "for geometric algebra existed in India before the classical period in Greece." (1978:323)

It is now generally understood that the so-called "Pythagorean theorem" was understood in ancient India,

and was in fact proved in Baudhayana's Shulva Sutra, a text dated to circa 600 BCE. (1978:323).

Knowledge of mathematics, and geometry in particular, was necessary for the precise construction of the complex Vedic altars, and mathematics was thus one of the topics covered in the brahmanas. This knowledge was further elaborated in the kalpa sutras, which gave more detailed instructions concerning Vedic ritual. Several of these treat the topic of altar construction. The oldest and most complete of these is the previously mentioned Shulva Sutra of Baudhaayana. As this text was composed about a century before Pythagoras, the theory that the Greeks were the source of Geometric algebra is untenable, while the hypothesis that India has been a source for Greek geometry, transmitted via the Persians who traded both with the Greeks and the Indians, looks increasingly plausible. On the other hand, it is quite possible that both the Greeks and the Indians developed geometry. Seidenberg has argued, in fact, that both seem to have developed geometry out of the practical problems involving their construction of elaborate sacrificial altars. (See Seidenberg 1962 and 1983)

Zero and the Place Value System

Far more important to the development of modern mathematics than either Greek or Indian geometry was the development of the place value system of enumeration, the base ten system of calculation which uses nine numerals and zero to represent numbers ranging from the most minuscule decimal to the most inconceivably large power of ten. This system of enumeration was not developed by the Greeks, whose largest unit of enumeration was the myriad (10,000) or in China, where 10,000 was also the largest unit of enumeration until recent times. Nor was it developed by the Arabs, despite the fact that this numeral system is commonly called the Arabic numerals in Europe, where this system was first introduced by the Arabs in the thirteenth century.

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Rather, this system was invented in India, where it evidently was of quite ancient origin. The Yajurveda Samhitaa, one of the Vedic texts predating Euclid and the Greek mathematicians by at least a millennium, lists names for each of the units of ten up to 10 to the twelfth power (paraardha). (Subbarayappa 1970:49)

Later Buddhist and Jain authors extended this list as high as the fifty-third power, far exceeding their Greek contemporaries, who lacking a system of enumeration were unable to develop abstract mathematical concepts.

The place value system of enumeration is in fact built into the Sanskrit language, where each power of ten is given a distinct name. Not only are the units ten, hundred and thousand (daza, zata, sahasra) named as in English, but also ten thousand, hundred thousand, ten million, hundred million (ayuta, laksa, koti, vyarbuda), and so forth up to the fifty-third power, providing distinct names where English makes use of auxiliary bases such as thousand, million, etc. Ifrah has commented that; by giving each power of ten an individual name, the Sanskrit system gave no special importance to any number. Thus the Sanskrit system is obviously superior to that of the Arabs (for whom the thousand was the limit), or the Greeks and Chinese (whose limit was ten thousand) and even to our own system (where the names thousand, million etc. continue to act as auxiliary bases). Instead of naming the numbers in groups of three, four or eight orders of units, the Indians, from a very early date, expressed them taking the powers of ten and the names of the first nine units individually. In other words, to express a given number, one only had to place the name indicating the order of units between the name of the order of units immediately immediately below it and the one immediately above it. That is exactly what is required in order to gain a precise idea of the place-value system, the rule being presented in a natural way and thus appearing self-explanatory. To put

it plainly, the Sanskrit numeral system contained the very key to the discovery of the place-value system. (2000:429)

As Ifrah has shown at length, there is little doubt that our place-value numeral system developed in India (2000:399-409), and this system is embedded in the Sanskrit language, several aspects of which make it a very logical language, well suited to scientific and mathematical reasoning. Nor did this system exhaust Indian ingenuity; as van Nooten has shown, Pingala, who lived circa the first century BCE, developed a system of binary enumeration convertible to decimal numerals, described in his *Chandahzaastra*. His system is quite similar to that of Leibniz, who lived roughly fourteen hundred years later. (See Van Nooten)

India is also the locus of another closely related and equally important mathematical discovery, the numeral zero. The oldest known text to use zero is a Jain text entitled the *Lokavibhaaga*, which has been definitely dated to Monday 25 August 458 CE. (Ifrah 2000:417-1 9)

This concept, combined by the place-value system of enumeration, became the basis for a classical era renaissance in Indian mathematics.

The Indian numeral system and its place value, decimal system of enumeration came to the attention of the Arabs in the seventh or eighth century, and served as the basis for the well known advancement in Arab mathematics, represented by figures such as al-Khwarizmi. It reached Europe in the twelfth century when Adelard of Bath translated al-Khwarizmi's works into Latin. (Subbarayappa 1970:49)

But the Europeans were at first resistant to this system, being attached to the far less logical roman numeral system, but their eventual adoption of this system led to the scientific revolution that began to sweep Europe beginning in the thirteenth century.

Luminaries of Classical Indian Mathematics**Aryabhata**

The world did not have to wait for the Europeans to awake from their long intellectual slumber to see the development of advanced mathematical techniques. India achieved its own scientific renaissance of sorts during its classical era, beginning roughly one thousand years before the European Renaissance. Probably the most celebrated Indian mathematicians belonging to this period was Aryabhata, who was born in 476 CE.

In 499, when he was only 23 years old, Aryabhata wrote his *Aaryabhattachiya*, a text covering both astronomy and mathematics. With regard to the former, the text is notable for its awareness of the relativity of motion. (See Kak p. 16)

This awareness led to the astonishing suggestion that it is the Earth that rotates the Sun. He argued for the diurnal rotation of the earth, as an alternate theory to the rotation of the fixed stars and sun around the earth (Pingree 1981:18).

He made this suggestion approximately one thousand years before Copernicus, evidently independently, reached the same conclusion.

With regard to mathematics, one of Aryabhata's greatest contributions was the calculation of sine tables, which no doubt was of great use for his astronomical calculations. In developing a way to calculate the sine of curves, rather than the cruder method of calculating chords devised by the Greeks, he thus went beyond geometry and contributed to the development of trigonometry, a development which did not occur in Europe until roughly one thousand years later, when the Europeans translated Indian influenced Arab mathematical texts.

Aryabhata's mathematics was far ranging, as the topics he covered include geometry, algebra, trigonometry. He also developed methods of solving quadratic and indeterminate

equations using fractions. He calculated π to four decimal places, i.e., 3.1416. (Pingree 1981:57) In addition, Aryabhata "invented a unique method of recording numbers which required perfect understanding of zero and the place-value system." (Ifrah 2000:419)

Given the astounding range of advanced mathematical concepts and techniques covered in this fifth century text, it should be of no surprise that it became extremely well known in India, judging by the large numbers of commentaries written upon it. It was studied by the Arabs in the eighth century following their conquest of Sind, and translated into Arabic, whence it influenced the development of both Arabic and European mathematical traditions.

Brahmagupta

Born in 598 CE in Rajasthan in Western India, Brahmagupta founded an influential school of mathematics which rivalled Aryabhata's. His best known work is the *Brahmasphuta Siddhanta*, written in 628 CE, in which he developed a solution for a certain type of second order indeterminate equation. This text was translated into Arabic in the eighth century, and became very influential in Arab mathematics. (See Kak p. 16)

Mahavira

Mahavira was a Jain mathematician who lived in the ninth century, who wrote on a wide range of mathematical topics. These include the mathematics of zero, squares, cubes, square-roots, cube-roots, and the series extending beyond these. He also wrote on plane and solid geometry, as well as problems relating to the casting of shadows. (Pingree 1981:60)

Bhaskar

Bhaskar was one of the many outstanding mathematicians hailing from South India. Born in 1114 CE

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in Karnataka, he composed a four-part text entitled the *Siddhanta Ziromani*. Included in this compilation is the *Bijaganita*, which became the standard algebra textbook in Sanskrit. It contains descriptions of advanced mathematical techniques involving both positive and negative integers as well as zero, irrational numbers. It treats at length the "pulverizer" (*kuttakaara*) method of solving indeterminate equations with continued fractions, as well as the so-called "Pell's equation" (*vargaprakrti*) dealing with indeterminate equations of the second degree. He also wrote on the solution to numerous kinds of linear and quadratic equations, including those involving multiple unknowns, and equations involving the product of different unknowns. (Pingree 1981, p. 64)

In short, he wrote a highly sophisticated mathematical text that preceded by several centuries the development of such techniques in Europe, although it would be better to term this a rediscovery, since much of the Renaissance advances of mathematics in Europe was based upon the discovery of Arab mathematical texts, which were in turn highly influenced by these Indian traditions.

Madhava

The Kerala region of South India was home to a very important school of mathematics. The best known member of this school Madhava (c. 1444-1545), who lived in Sangamagrama in Kerala. Primarily an astronomer, he made history in mathematics with his writings on trigonometry. He calculated the sine, cosine and arctangent of the circle, developing the world's first consistent system of trigonometry. (See Hayashi 1997:784-786) He also correctly calculated the value of π to eleven decimal places. (Pingree 1981:490)

This is by no means a complete list of influential Indian mathematicians or Indian contributions to mathematics, but rather a survey of the highlights of what is, judged by any fair, unbiased standard, an illustrious tradition,

important both for its own internal elegance as well as its influence on the history of European mathematical traditions. The classical Indian mathematical renaissance was an important precursor to the European renaissance, and to ignore this fact is to fail to grasp the history of latter, a history which was truly multicultural, deriving its inspiration from a variety of cultural roots.

There are in fact, as Frits Staal has suggested in his important (1995) article, "The Sanskrit of Science", profound similarities between the social contexts of classical India and renaissance Europe. In both cases, important revolutions in scientific thought occurred in complex, hierarchical societies in which certain elite groups were granted freedom from manual labour, and thus the opportunity to dedicate themselves to intellectual pursuits. In the case of classical India, these groups included certain brahmins as well as the Buddhist and Jain monks, while in renaissance Europe they included both the monks as well as their secular derivatives, the university scholars.

Why, one might ask, did Europe take over thousand years to attain the level of abstract mathematics achieved by Indians such as Aryabhata? The answer appears to be that Europeans were trapped in the relatively simplistic and concrete geometrical mathematics developed by the Greeks. It was not until they had, via the Arabs, received, assimilated and accepted the place-value system of enumeration developed in India that they were able to free their minds from the concrete and develop more abstract systems of thought. This development thus triggered the scientific and information technology revolutions which swept Europe and, later, the world.

The role played by India in the development is no mere footnote, easily and inconsequentially swept under the rug of Eurocentric bias. To do so is to distort history, and to deny India one of its greatest contributions to world civilization.

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Vedic mathematics is a system of mental calculation developed by Shri Bharati Krishna Tirthaji which he claimed he had based on a lost appendix of Atharvaveda, an ancient text of the Indian teachings called Veda.

It has some similarities to the Trachtenberg system in that it speeds up some arithmetic calculations. It claims to have applications to more advanced mathematics, such as calculus and linear algebra. The system was first published in the book *Vedic Mathematics* ISBN 8120801644 in 1965. The system has since been developed further and there have been several other books released.

Criticism

Critics have questioned whether this subject deserves the name Vedic or indeed mathematics. They point to the lack of evidence of any sutras from the Vedic period consistent with the system, the inconsistency between the topics addressed by the system (such as decimal fractions) and the known mathematics of early India, the substantial extrapolations from a few words of a sutra to complex arithmetic, and the restriction of applications to convenient cases.

They further say that such arithmetic as is speeded up by application of the sutras can be performed on a computer or calculator anyway, making their knowledge rather irrelevant in the modern world.

They are also worried that it deflects attention from genuine achievements of ancient and modern Indian mathematics and mathematicians, and that its promotion by Hindu nationalists may damage mathematics education in India. Actual Indian mathematicians, at leading institutions such as TIFR or IISc, largely share these opinions.

The system is based upon sixteen formulas and their corollaries, some of which are described below.

All from Nine and the Last from Ten

Corollary 1: Whatever the extent of its deficiency, lessen it still further to that very extent; and also set up the square of that deficiency.

For instance, in computing the square of 9 we go through the following steps:

1. The nearest power of 10 to 9 is 10. Therefore, let us take 10 as our base.
2. Since 9 is 1 less than 10, decrease it still further to 8. This is the left side of our answer.
3. On the right hand side put the square of the deficiency, that is 12. Hence the answer is 81.
4. Similarly, $8^2 = 64$, $7^2 = 49$.
5. For numbers above 10, instead of looking at the deficit we look at the surplus. For example:
and so on.

This is based on the identities $(a + b)(a - b) = a^2 - b^2$ and $(a + b)^2 = a^2 + 2ab + b^2$.

By One More than the One Before

The proposition "by" means the operations this formula concerns are either multiplication or division. [In case of addition/subtraction proposition "to" or "from" is used.] Thus this formula is used for either multiplication or division. It turns out that it is applicable in both operations.

An interesting application of this formula is in computing squares of numbers ending in five. Consider:

$$35 \times 35 = (3 \times (3 + 1)), 25 = 12,25$$

The latter portion is multiplied by itself (5 by 5) and the previous portion is multiplied by one more than itself (3 by 4) resulting in the answer 1225.

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This is a simple application of $(a + b)^2 = a^2 + 2ab + b^2$ when $a = 10c$ and $b = 5$, i.e.

$$(10c + 5)^2 = 100c^2 + 100c + 25 = 100c(c + 1) + 25.$$

It can also be applied in multiplications when the last digit is not 5 but the sum of the last digits is the base (10) and the previous parts are the same. Consider:

$$37 \times 33 = (3 \times 4), 7 \times 3 = 12, 21$$

$$29 \times 21 = (2 \times 3), 9 \times 1 = 6, 09$$

This uses $(a + b)(a - b) = a^2 - b^2$ twice combined with the previous result to produce:

$$(10c + 5 + d)(10c + 5 - d) = (10c + 5)^2 - d^2$$

$$= 100c(c + 1) + 25 - d^2$$

$$= 100c(c + 1) + (5 + d)(5 - d).$$

We illustrate this formula by its application to conversion of fractions into their equivalent decimal form. Consider fraction $1/19$. Using this formula, this can be converted into a decimal form in a single step. This can be done by applying the formula for either a multiplication or division operation, thus yielding two methods.

METHOD 1: USING MULTIPLICATIONS

$1/19$, since 19 is not divisible by 2 or 5, the fractional result is a purely circulating decimal. (If the denominator contains only factors 2 and 5 is a purely non-circulating decimal, else it is a mixture of the two.)

So we start with the last digit

1

Multiply this by "one more", that is, 2 (this is the "key" digit from Ekadhikena)

21

Multiplying 2 by 2, followed by multiplying 4 by 2

421 $\rightarrow$ 8421

Now, multiplying 8 by 2, sixteen

68421

1 ← carry

multiplying 6 by 2 is 12 plus 1 carry gives 13

368421

1 ← carry

Continuing

7368421 → 47368421 → 947368421

1

Now we have 9 digits of the answer. There are a total of 18 digits (= denominator ? numerator) in the answer computed by complementing the lower half:

052631578

947368421

Thus the result is .052631578,947368421

METHOD 2: USING DIVISIONS

The earlier process can also be done using division instead of multiplication. We divide 1 by 2, answer is 0 with remainder 1

.0

Next 10 divided by 2 is five

.05

Next 5 divided by 2 is 2 with remainder 1

.052

next 12 (remainder,2) divided by 2 is 6

.0526

and so on.

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As another example, consider $1/7$, this same as $7/49$ which as last digit of the denominator as 9. The previous digit is 4, by one more is 5. So we multiply (or divide) by 5, that is,

$\dots 7 \Rightarrow 57 \Rightarrow 857 \Rightarrow 2857 \Rightarrow 42857 \Rightarrow 142857 \Rightarrow .142,857$
(stop after 7 $\Rightarrow$ 1 digits)

3 2 4 1 2

Vertically and Crosswise

This formula applies to all cases of multiplication and is very useful in division of one large number by another large number.

Transpose and Apply

This formula complements "all from nine and the last from ten", which is useful in divisions by large numbers. This formula is useful in cases where the divisor consists of small digits. This formula can be used to derive the Horner's process of Synthetic Division.

When the Samuccaya is the Same, that Samuccaya is Zero

This formula is useful in solution of several special types of equations that can be solved visually. The word samuccaya has various meanings in different applications. For instance, it may mean a term which occurs as a common factor in all the terms concerned. A simple example is equation " $12x + 3x = 4x + 5x$ ". Since " x " occurs as a common factor in all the terms, therefore, $x = 0$ is a solution. Another meaning may be that samuccaya is a product of independent terms. For instance, in $(x + 7)(x + 9) = (x + 3)(x + 21)$, the samuccaya is $7 \times 9 = 3 \times 21$, therefore, $x = 0$ is a solution. Another meaning is the sum of the denominators of two fractions having the same numerical numerator, for example: $1/(2x - 1) + 1/(3x - 1) = 0$ means $5x - 2 = 0$.

Yet another meaning is "combination" or total. This is commonly used. For instance, if the sum of the numerators and the sum of denominators are the same then that sum is zero. Therefore,

$$\text{Therefore, } 4x + 16 = 0 \text{ or } x = -4.$$

This meaning ("total") can also be applied in solving quadratic equations. The total meaning can not only imply sum but also subtraction. For instance when given $N_1/D_1 = N_2/D_2$, if $N_1 + N_2 = D_1 + D_2$ (as shown earlier) then this sum is zero. Mental cross multiplication reveals that the resulting equation is quadratic (the coefficients of x^2 are different on the two sides). So, if $N_1 - D_1 = N_2 - D_2$ then that samuccaya is also zero. This yields the other root of a quadratic equation.

Yet interpretation of "total" is applied in multi-term RHS and LHS. For instance, consider

$$\text{Here } D_1 + D_2 = D_3 + D_4 = 2x - 16.$$

$$\text{Thus } x = 8.$$

There are several other cases where samuccaya can be applied with great versatility. For instance "apparently cubic" or "biquadratic" equations can be easily solved as shown below:

$$(x - 3)^2 + (x - 9)^3 = 2(x - 6)^3.$$

$$\text{Note that } x - 3 + x - 9 = 2(x - 6).$$

$$\text{Therefore } (x - 6) = 0 \text{ or } x = 6.$$

(remark by different author: Note also that:

$$(6 - 3)^2 + (6 - 9)^3 = 3^2 - 3^3 = 9 - 27 = -18,$$

$$\text{whereas } 2(6 - 6)^3 = 0$$

Thus: $x = 6$ is NOT a solution!)

(Another note. This example does work if one considers all exponents to be cubic, in which case $(6 - 3)^3 + (6 - 9)^3$ will give $27 - 27$.)

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Consider

$$\text{Observe: } N_1 + D_1 = N_2 + D_2 = 2x + 8.$$

Therefore, $x = -4$.

This formula has been extended further.

If One is in Ratio, the Other One is Zero

This formula is often used to solve simultaneous simple equations which may involve big numbers. But these equations in special cases can be visually solved because of a certain ratio between the coefficients. Consider the following example:

$$6x + 7y = 8$$

$$19x + 14y = 16$$

Here the ratio of coefficients of y is same as that of the constant terms. Therefore, the "other" is zero, i.e., $x = 0$. Hence the solution of the equations is $x = 0$ and $y = 8/7$.

(alternatively:

$19x + 14y = 16$ is equivalent to:

$$(19/2)x + 7y = 8.$$

Thus it is obvious that x has to be zero, no ratio needed, just div by²!)

This formula is easily applicable to more general cases with any number of variables. For instance

$$ax + by + cz = a$$

$$bx + cy + az = b$$

$$cx + ay + bz = c$$

which yields $x = 1, y = 0, z = 0$.

A corollary says By addition and by subtraction. It is applicable in case of simultaneous linear equations where the x - and y -coefficients are interchanged. For instance:

$$45x - 23y = 113$$

$$23x - 45y = 91$$

By addition:

$$68x - 68y = 204$$

$$\Rightarrow 68(x - y) = 204$$

$$\Rightarrow x - y = 3.$$

By subtraction:

$$22x + 22y = 22$$

$$\Rightarrow 22(x + y) = 22$$

$$\Rightarrow x + y = 1.$$

5

Science and Technology in Epics

How much hi-tech and science did our forefathers really know? On the one hand, there are emphatic claims that they knew everything that modern science has taught us today and will teach us in the years to come. On the other, there are skeptics who would associate the dawn of science and technology with the West, dating back no more than a few centuries and would thus deny any knowledge to oriental ancients.

The arguments "for" fall into three categories. The first line of argument takes to our puranas and the epics. In the Ramayana, we have the Pushpak Vimana which clearly establishes the existence of helicopters and aeroplanes. What better example of a laser-guided missile than the Shakti with which Karna killed Ghatotkacha in the Mahabharata? And the instant beaming of a living body across vast distances that you see in the futuristic Star Trek was routinely practised by our devas and rishis.

Ancient Work

The next line of argument invokes ancient works like the Brihad Vimanshastra which describes different flying vehicles and methods of constructing them. Then there are interpretations of Vedic Richas to show that they contain

details of the nuclear fusion that powers the Sun. There is Vedic mathematics which is claimed to contain advanced mathematical concepts and methods. It has often been said that the philosophers of the Vedic and later times knew about quantum mechanics, gauge theory, unified field theory and so on which the theoretical physicists of today are grappling with.

The third argument brings forth examples of old tests like the Vedanga Jyotisha attributed to Lagadha which has astronomical discussions relating to the solar system, the Shalbhya Sutra containing geometric ideas, and the Charaka and Sushruta Samhitas which contain ideas on medicine and surgery. Surely these demonstrate that some science was known and practised in the ancient times.

You don't need a lawyer to tell you that a case is weakened if some of its arguments are demonstrably refutable or are basically ill-formulated. The first two lines of arguments fall into in this category.

Thus to convince a scientist it is not sufficient to refer to descriptions of the epics and puranas. Absorbing and imaginative though they are they are far cry from what constitutes a technical manual.

Imagine that thousands of years later our descendents recover a cassette of Star Trek from an archaeological excavation and manage to view it. Are they justified in concluding on the basis of what they see that we ourselves were in possession of the kind of technology shown there? Therefore, we can credit our ancients with giant leaps of imagination to have produced those works...but no more.

For, if we treat these descriptions as of reality then we run into trouble. In our epics and puranas we see no description of electrical power that is the first significant step of high technology. Why do we find no reference to even the capital cities of Dasharatha or Dhritrashtra enjoying

this amenity? Remember that of the four fundamental forces of nature the electrical one has the largest variety of manifestations in nature and should be the first one to be extensively studied, understood and applied.

I had studied the Brihad Vimanshastra and failed to find in it a technical description of aeronautics. For example, a shastra could contain a basic framework of rules governing flying objects which then just translated into technical reality. Basic principles of flying like what is today known as Bernoulli's theorem, the idea of life and streamlines should be described.

Even assuming that the ancients somehow bypassed these basics through some yogic insight no available to modern scientist, the actual description of construction is also vague and certainly cannot be used for making a real aircraft.

Vedic Mathematics

Much is made of Vedic mathematics. Leaving aside the claims and counterclaims about its adjective "Vedic" let us look at the mathematics part. If it is looked upon as a system of carrying out quick numerical manipulation it has its merits. But as any mathematician of distinction will tell you, that does not constitute advanced mathematics.

I love Sanskrit as a language and have enjoyed reading its classics. But its very verstality proves a stumbling block in interpreting ancient descriptions and with Vedic Sanskrit (which I confess, I cannot readily follow) the problem of interpretation is even more tricky.

It is easy for the interpreter to read into a statement the meaning he or she wishes to read. This probably explains the post facto nature of interpretations: only after modern science makes a discovery do claims appear that it is contained in such an such ancient tome. In many cases, the

antiquity of the statement may also be questionable in the sense that later writers inserted their own ideas into the old works. Simply because a statement occurs in Sanskrit does not mean it is ancient!

Arguments in the third category, however, are worth taking note of and deserve further scholarly studies. For they are clearly documented, can be interpreted without "personal" ambiguities and do tell us that our ancient civilizations did have thinkers and experimenters of high quality.

Thus Lagadha should be studied with as much care as his (much) later descendents like Aryabhatta (fifth century A.D.) and Bhaskara (12th century A.D.). The works ascribed to Sushruta and Charaka, unlike speculations described above, are experimentally testable. The legacy of yoga as a discipline of mind and body is indisputable.

Old Text

Our old texts are often examined for astronomical events like eclipses, comets and conjunctions. Another useful and hitherto unexplored line of investigation is supernovae. A supernova is an exploding star: an event which can be spectacular if it takes place in our galaxy.

The Chinese and Japanese astronomers have recorded an event that was seen on July 4, 1054... when the exploding star brightened so much as to be visible during daylight. Surely a spectacular even like that would not have gone unnoticed on our subcontinent. With the glorious tradition from Aryabhatta to Bhaskara continuing at that time it is worth looking for records wherever they may be. A finding will be of enormous importance to astronomical archives.

Let me end with an example from social sciences. In his book *Labour Management* the late R.G. Rajwade has an appendix on *Shukra Neeti* written mostly in Gupta period.

Shukra had laid down precise rules for the employee and the employer, spelling out details on the type of wages, periods of payment, grades of employment, fair wages, industrial disputes, leave rules, sickness benefits, pension, provident fund and family allowance, bonus etc.

For example, Shukra recommends that an employee who has served for forty years should be paid a pension half his wages throughout his life. For bonus he recommends one eighth of the employee's salary.

I mention this example to illustrate the fact that our ancients could be precise when they chose to be so. Let us stress examples of precision which is the hallmark of science rather than ambiguous personalised interpretations of what they said. Only then will we do justice to the achievements of our ancients.

World of Mathematics: The Indian Contribution

The Contribution to Mathematics by India can be divided into ten categories:

1. Zero and the place-value notation for numbers
2. Vedic Mathematics and arithmetical operations
3. Geometry of the Sulba Sutras
4. Jaina contribution to Fundamentals of numbers
5. The anonymous Bakshali manuscript
6. Astronomy
7. Classical contribution to Indeterminate Equations and Algebra
8. Indian Trigonometry
9. Kerala contribution to Infinite Series and Calculus.
10. Modern Contribution: Srinivasa Ramanujan onwards

Section 1: Zero and the Place-Value Notation

The number zero is the subtle gift of the Hindus of antiquity to mankind. The concept itself was one of the most significant inventions in the ascent of Man for the growth of culture and civilization. To it must be credited the enormous usefulness of its counterpart, the place value system of expressing all numbers with just ten symbols. And to these

two concepts we owe all the arithmetic and mathematics based upon them, the great ease which it has lent to all computations for two millenia and the binary system which now lies at the foundation of communicating with computers. Already in the first three centuries A.D., the Hindu ancients were using a decimal positional system, that is, a system in which numerals in different positions represent different numbers and in which one of the ten symbols used was a fully functional zero.

They called it 'Sunya'. The word and its meaning 'void' were obviously borrowed from its use in philosophical literature. Though the Babylonians used a special symbol for zero as early as the 3rd century B.C., they used it only as a place holder; they did not have the concept of zero as an actual value. It appears the Maya civilisation of South America had a zero in the first century A.D. but they did not use it in a fixed base system. The Greeks were hampered by their use of letters for the numbers. Before zero was invented, the art of reckoning remained an exclusive and highly skilled profession. It was difficult to distinguish, say, 27, 207, 270, 2007, because the latter three were all written 2 7, with a 'space' in between. The positional system is not possible in the Roman numeral system which had no expression or symbol for zero. A number, say, 101,000, would have to be written only by 101 consecutive M's. The Egyptians had no zero and never reached the idea of expressing all numbers with ten digits. The mathematical climate among the Hindus, however, was congenial for the invention of zero and for its use as the null-value in all facets of calculation, due to four factors:

1. A notation for powers of 10 upto the power 17 was already in use even from vedic times. Single words have been used to denote the powers of the number 10. The numbers one, ten, hundred, thousand, ten thousand, ... are given by the sequence of words in the list:

eka, dasa, Sata, sahasra, ayuta, laksha, prayuta, koTi, arbuda, abja, kharva, nikharva, mahA-padma, Sankha, jaladhi, antya, mahASankha, parArdha. Thus the decimal system was in the culture even in the early part of the first millenium B.C.. The Yajurveda, in its description of rituals and the mantras employed therein, the Mahabharata and the Ramayana in their descriptions of statistics and measurements, used all these words, with total abandon.

2. Counting boards with columns representing units and tens were in use from very ancient times. The numberless content of an empty column in course of time was symbolized to be 'nothing'.
3. The thriving activity in astrology, astronomy, navigation and business during the first few centuries A.D. naturally looked forward to a superior numerical system that lent itself to complicated calculations.
4. Distinct symbols for the numbers 1 to 9 already existed and the counting system used the base 10 in all its secular, religious and ritual activities. Compare this with the Babylonian numeration which had only three figures, one for 1, one for 10, and one for 100, so that a number, say, 999, would require 27 symbols, namely, nine of each of the symbols.

Of these, the first and fourth factors are probably unique to Hindu culture and contributed most to the thought process that led to the decimal place value notation as well as zero having a value. When exactly the invention of this most modest of all numerals took place, we do not know. The first time it reached Europe was during the Moorish invasion of Spain around 700 A.D. Later, when massive Latin translations of books from Baghdad took place around the close of the first millenium A.D. the concept was found in an arithmetic book dated 820 A.D., by Muhammad Ibn

Musa al-Khouarizmi who explained the whole system in great detail. The Arabs themselves had no number system of their own. It was the Hindu system he was explaining. That is why the numerals are called the Indo-Arabic numerals even today. It is however a misnomer to call them so; they are already found on the Rock Edicts of Ashoka (256 B.C.). In spite of its being so crucial to our living, it took centuries for the western world to appreciate and incorporate this most valuable numeral, zero, in their recording of accounts or in scholarly writings. For by the time 'Zero' reached the West, the Dark Ages of the western world had begun. There are traces, however, of its knowledge in Spain in the tenth century A.D. But the final breakthrough of the introduction to the West was by Leonardo of Pisa, through his popular text *Liber Abaci*, 1202 A.D. The first European book (in French) that used the zero appeared in 1275.

How the Sunya of the Hindus became the Zero of the modern world is interesting. The 'Sunya' of Sanskrit became the Arabic 'sifr' which means empty space. In medieval Latin it manifested as 'ciphra', then in middle English as 'siphre', in English as 'cypher' and in American as 'cipher'. In the middle ages, the word 'ciphra' evolved to stand for the whole system. In the wake of this general meaning, the Latin 'zephirum' came to be used to denote the Sunya. And that entered English finally as 'zero'. In medieval Europe some countries banned the positional number system, along with zero, brought by the Arabs whom they considered as heathens. So they took Sunya to be a creation of the devil! As a result 'ciphra' came to mean a secret code. From this came 'deciphering', the resolution of a code!

Section 2. Vedic Mathematics and Arithmetical Operations

Vedic Mathematics provides an original and refreshing approach to subjects which are usually dismissed as mechanical and tedious. Bharati Krishna Tirtha who

published his reconstruction of Vedic Mathematics in 1965, maintains that there are 16 aphorisms and 13 secondary aphorisms which forms his base of the so-called Vedic Mathematics. Though the origins of Vedic Mathematics have not yet been historically established, if nothing else, it provides tremendous insights into the place-value system of numbers without which it would not work. It is amazing that Vedic Mathematics does not require of cramming of multiplication tables beyond 5×5 . One can improvise all the necessary multiplication tables for oneself and with the aid of the relevant Vedic formulae get the required products very easily, speedily, and correctly, almost immediately. The formulae can be used to evaluate determinants, solve simultaneous linear equations, evaluate logarithms and exponentials. Vedic Mathematics recognises that any algebraic polynomial may be expressed in terms of a positional notation without specifying the base. The same algorithmic scheme as applied to arithmetical operations will easily apply to algebraic problems. And this brings it to the Modern Algebra of Polynomials. It is difficult, in a historical introduction like this to get into the details of Vedic Mathematics. Suffice it to say that with today's over-dependence on calculators for even simple arithmetical computations, the Vedic methods have great pedagogical value and, through their revival, the skills of mental arithmetic may not be lost for posterity.

Section 3. Geometry of the Sulba Sutras

Hailing from the times of the Vedas, the ritual literature which gave directions for constructing sacrificial fires at different times of the year dealt with the their measurement and construction in a systematic and logical way, thus giving rise to the Sulba Sutras. The construction of altars (vedi) and the location of sacrificial fires had to conform to clearly laid down instructions about their shapes and areas in order that they may be effective instruments of sacrifice. The

Sulba Sutras provide such instructions for two types of ritual - one for worship at home and one for communal worship. The instructions were mainly for the benefit of craftsmen laying out and building the altars. Bodhayana, Apastamba and Katyayana who have recorded these Sulbasutras were not only priests in the conventional sense but must have been craftsmen themselves. The earliest of them, The Bodhayana Sutras, in three chapters, (800 - 600 B.C.) contains a general statement of the Pythagorean theorem, an approximation procedure for obtaining the square root of two correct to five decimal places and a number of geometric constructions. These latter include an approximate squaring the circle, and construction of rectilinear shapes whose area is equal to the sum or difference of areas of other shapes. The Bodhayana version of the Pythagorean theorem states as follows:

The rope which is stretched across the diagonal of a square produces an area double the size of the original square.

It is therefore in the fitness of things that the Pythagorean theorem of Mathematics may be renamed as the Bodhayana theorem. The other sutras are two centuries later but all of them are prior to Panini of the fourth century B.C. The geometry arising from these sutras give several geometric constructions. Some of these are:

1. To merge two equal or unequal squares to obtain a third square.
2. To transform a rectangle into a square of equal area
3. Squaring a circle and circling a square (approximately)

A remarkable achievement was the discovery of a procedure for evaluating square roots to a high degree of approximation. The square root of two is obtained as

1.4142156 ...

the true value being 1.414213... . The fact that such procedures were used successfully by the Sulbasutra geometers to operations with other irrational numbers, is clear proof for negating the western-held opinion that the Sulba sutra geometers borrowed their methods from the Babylonians. The latter's calculation of the square root of two is an isolated instance and further they used the sexagesimal notation for numbers. The achievement of geometrical constructs in Indian mathematics reached its peak later when they arrived at the construction of Sriyantra, which is a complicated diagram, consisting of nine interwoven isosceles triangles, four pointing upwards and four pointing downwards. The triangles are arranged in such a way that they produce 43 subsidiary triangles, at the centre of the smallest of which there is a big dot called the binḍu. The difficult problem is to construct the diagram in such a way that all the intersections are correct and the vertices of the largest triangles fall on the circumference of the enclosing circle. In all cases the base angles of the largest triangles is about 51.5 degrees. This has connections with the two most famous irrational numbers of Mathematics. The quantity, called the golden ratio, has remarkable mathematical properties and is almost a semi-mystical number.

Section 4. Jaina Contribution to Fundamentals of Numbers

By the time of the Jains, the role of rituals in the development of mathematics declined and mathematics began to be pursued also for its own sake. The Jains had a fascination for large numbers. Their definitions of the various types of infinities they comprehended are sophisticated, though lacking in mathematical precision. But it must be said to their credit that they were the first, in the chronology of scientific thinking, to have recognised that all infinities were not the same or equal. In fact this

idea was established in the mathematical world only in the latter half of the nineteenth century when Cantor initiated his theory of sets.

The Jains were also aware of the theory of indices, though they did not have the modern notation or any convenient notation for the same. Calling the successive squares and square roots as the first, the second, etc. they make the following statement: The first square root multiplied by the second square root is the cube of the second square root. In modern notation this is nothing but the identity in the theory of indices:

$$a^{1/2} \times a^{1/4} = (a^{1/4})^3$$

They have several such rules for working with powers of a number. They also seem to have had an idea of the logarithm of a number though they don't seem to have put them to practical use in calculation. Another favourite topic with them was the study of permutations and combinations. They had also a great interest in sequences and progressions developed out of their philosophical theory of cosmological structures. A Jain canonical text entitled *Triloka prajnapati* has a very detailed treatment of arithmetic progressions.

Section 5: The Anonymous Bakshali Manuscript

This manuscript was discovered in 1881 A.D. near a village called Bakshali. It is written in an old form of Sanskrit on seventy leaves of birch bark. It is probably a copy of a manuscript composed in the early centuries A.D.

It is a handbook of rules and illustrative examples together with solutions, all mainly on arithmetic and algebra.

Fractions, Square roots, Profit and Loss, Interest, Rule of Three, Approximation to surds, Simple equations as well as Simultaneous equations, Quadratic equations, Arithmetic and Geometric Progressions -- all these are covered. Very unusually in the entire history of Ancient Indian

mathematics, the subject matter is organised in a sequence: first, a rule or a sutra; then a relevant example in word form; the same in notational form; then the solution and finally the demonstration or the proof.

Here for the first time in the history of world mathematics, the Rule of Three is stated in its abstract form. It was from here that the rule was taken to Europe by the Arabs and it was then known as the Golden Rule. It became very popular in Europe after the Renaissance. The rule goes as follows:

If p yields f what will i yield?

Here p stands for pramAna, f for phala and i for icchA.

Here p and i are of the same denomination and f is of a different denomination.

Write p, f, i in that order. Multiply the middle quantity by the last quantity and divide by the first. The result is fi / p .

The first appearance of indeterminate equations is in the Bakshali mss. This marks the beginning of the continued work on indeterminate equations in India. Such an interest, though originally generated from the demands made by astronomical calculations, was also pursued for its own sake, sometimes even for recreational purposes. The ease with which the place value system is worked in the Bakshali mss. suggests that the system predates the mss. by probably a few centuries.

Section 6. Astronomy

The contribution to Astronomy by ancient Indians is so great that it does not befit it to include it as one of the contributions of Indian Mathematics to the rest of the world. It needs a separate forum all for itself. We shall leave it right there except to add a note on the ancient contribution to the

problem of telling time at night by a look at the stars on the meridian. This part is usually not emphasized.

The ancients of India have passed on to us 27 mathematical formulae coded in the Sanskrit language, but not very difficult to remember. In fact, very possibly it has mostly come down to us by oral transmission from generation to generation. For instance, the formula

krittika simhe kaya

says that if you see the asterism krittikA (Pleides, in modern terminology) on the meridian, that is the time the Leo (= simha) constellation (of the zodiac) has risen above the horizon by an amount indicated by the word: kAyA. This latter word interpreted in katapayA sankhyA, which is the notation used by astronomers, astrologers and mathematicians to represent numbers, means in this context that the amount of Leo above the (Eastern) horizon is 27 minutes of time.

From this and the known position of the Sun on the date in question, one mentally calculates the time of night. On November 7 for example, the Sun is in the middle of Scorpio. So if you see krittikA on the meridian it means Leo has risen 27 minutes before and this means the Sun is behind by

93m (remaining portion of simha)

+ 2h (full portion of kanyA)

+ 2h (full portion of tulA)

+ 60m (half portion of vrischika)

that is 6 hours 33 minutes. In other words it is 6h 33m before sunrise. So it is 11-27 P.M.

Suffice it to say these beautiful formulae constitute an intellectual marvel put to the most mundane use. Never perhaps was so much achieved with so little so early in the Ascent of Man.

Section 7. Classical Contribution to Indeterminate Equations and Algebra

The apex of Mathematical achievement of ancient India occurred during the so-called classical period of Indian Mathematics. The great names are: Aryabhatta I (b.476 A.D.); Brahmagupta (b.598 A.D.); Bhaskara I (circa 620 A.D.); Mahavira (circa 850 A.D.); Sridhara (circa 900 A.D.); Bhaskara II (b.1114 A.D.); Nilakantha Somayaji (1445 - 1545 A.D.).

Aryabhatta wrote the famous *Aryabhatiyam* which is an exhaustive exposition of Astronomy. In addition he gave a unique method of representing large numbers by word forms. He systematized all the knowledge of astronomy and mathematics prior to him. The first one in Indian mathematics to give the formula for the area of a triangle was Aryabhatta. Several results on Triangles and circles and on Progressions, algorithm for finding cube roots, approximation of π , all these give him a unique position in the development of mathematics. Aryabhatta ushered in a Renaissance in Indian Mathematics and Astronomy, that resulted in a remarkable flourishing of science and technology in India. Aryabhatta, for the first time, secularised mathematics and astronomy in India and established these as intellectual disciplines in their own right. Excellent commentators followed Aryabhatta and to them are due several modifications and applications, explanation of subtle points, and finally, the proofs of results embodied in the sutras of the Master.

Bhaskara I takes a large share of the credit of explaining the too brief and aphoristic statements of Aryabhatta. On the important topic of indeterminate equations the Kuttaka method was introduced by Aryabhatta and elucidated by Bhaskara I.

Brahmagupta is generally known as the Indian mathematician par excellence. His monumental work

Brahma SiddhAnta has 24 chapters of which the latter 14 contain original results on arithmetic algebra and on astronomical instruments. The 12th chapter is on mensuration. The 18th chapter is on Kuttaka. Among his famous results are those on rational right-angled triangles, and cyclic quadrilaterals. He is the earliest one, in the history of world mathematics, to have discussed cyclic quadrilaterals. There is every reason for us to name cyclic quadrilaterals as Brahmagupta Quadrilaterals. It was partly through a translation of Brahma-siddhAnta that the Arabs became aware of Indian astronomy and mathematics.

Bhaskara II's famous work SiddhAnta Siromani has four parts of which the first two are Mathematics and the latter two are astronomy. The first part, LilAvati is an extremely popular text dealing with arithmetic, algebra, geometry and mensuration. The second part, BIjaganitam is a treatise on Advanced Algebra. It contains problems on determining unknown quantities, evaluating surds and solving simple and quadratic equations.

The sheer ingenuity and versatility of Brahmagupta's approach to indeterminate equations of the second degree of the form

$$N x^2 + 1 = y^2$$

is the climax of Indian work in this area. Bhaskara II's cakravAla method to solve such equations is world-famous. By using this powerful method he solved, as one example, the above equation with $N = 61$ and gave the least integral solution as

$$x = 226153980 \text{ and } y = 1766319049.$$

The famous French mathematician, Fermat, in 1657 A.D. proposed this equation with $N = 61$ for solution as a challenge to his contemporaries. None of them succeeded in solving the equation in integers. It was not until 1767 A.D. that the western world through Euler, by Lagrange's method

of continued fractions, had a complete solution to such types of equations, wrongly called Pell's equation by Euler. But the very same equation, though coincidentally, was completely solved by Bhaskara II five hundred years earlier.

The problem of determining integer solutions of such equations is called Diophantine Analysis after the Greek Mathematician Diophantus (3rd cen. A.D.). As soon as one finds a non-trivial solution (that is, other than the obvious solution $x = 0, y = 1$) an infinite number of new solutions can be found by repeated application of the Principle of Compositions, known as Brahmagupta's Bhavana Principle. It is Bhaskara's cakravala method that makes the decisive step in determining a non-trivial solution. Under these circumstances it is appropriate to designate these equations as the Brahmagupta-Bhaskara equations.

Before we leave this topic it is important to mention Srinivasa Ramanujan, the 20th century genius, who revelled in such problems - namely, to determine the possible cases in which a number can be broken up into two or more equal sums of like or unlike powers or more generally to solve intermediate problems in rational numbers.

Bhaskara II introduces also the notion of instantaneous motion of planets. He clearly distinguishes between *sthUla gati* (average velocity) and *sUkshma gati* (accurate velocity) in terms of differentials. He also gave formulae for the surface area of a sphere and its volume, and volume of the frustum of a pyramid. Suffice it to say that his work on fundamental operations, his rules of three, five, seven, nine and eleven, his work on permutations and combinations and his handling of zero all speak of a maturity, a culmination of five hundred years of mathematical progress.

Section 8. Indian Trigonometry

Though Trigonometry goes back to the Greek period, the character of the subject started to resemble modern form

only after the time of Aryabhatta. From here it went to Europe through the Arabs and went into several modifications to reach its present form.

In ancient times Trigonometry was considered a part of astronomy. Three functions were introduced: *jya*, *kojya* and *ukramajya*. The first one is $r \sin$ where r is the radius of the circle and is the angle subtended at the centre. The second one is $r \cos$ and the third one is $r (1 - \cos)$. By taking the radius of the circle to be 1, we get the modern trigonometric functions. Various relationships between the sine of an arc and its integral and fractional multiples were used to construct sine tables for different arcs lying between 0 and 90.

Section 9: Kerala Contribution to Infinite Series and Calculus

Kerala mathematicians produced rules for second order interpolation to calculate intermediate sine values. The Kerala mathematician Madhava may have discovered the sine and cosine series about three hundred years before Newton. In this sense we may consider Madhava to have been the founder of mathematical analysis. Madhava (circa 1340 - 1425 A.D.) was the first to take the decisive step from the finite procedures of ancient Indian mathematics to treat their limit-passage to infinity. His contributions include infinite-series expansions of circular and trigonometric functions and finite-series approximations. His power series for and for sine and cosine functions is referred to reverentially by later writers.

Many later discoveries in European mathematics (for example, the Gregory series for the inverse tangent) were anticipated by Kerala astronomer-mathematicians. Nilakantha was mainly an astronomer but his *Aryabhatiya bhashya* and *tantra-sangraha* contain work on infinite-series expansions, problems of algebra and spherical geometry.

Section 10. Modern Contribution: Srinivasa Ramanujan Onwards

The second decade of the 20th century compulsorily turned the attention of the mathematical world to India and the Number Theory genius, Srinivasa Ramanujan. The ideas and innovative genius of Ramanujan have not been surpassed ever before or even 100 years after him. His birth, his super-activity in Madras and Cambridge, his glorious rise to international fame and unfortunate death—all happened almost in a flash. But ever since, India has remained on the mathematical map of the world more and more prominently. The 20th century saw the growth of Mathematics in India in the same style, though not in the same speed, the western world has been taking the race for the quest of mathematical knowledge. May we hope that the new millenium will see not only further speed in the acquisition and spread of mathematical knowledge by Indian-born mathematicians, but also further depth in the intensity and relevance of that knowledge, true to its traditions which are at least three millenia old .

Sacred Geometry

Details on this page is an extraction of some of the more interesting facts that have made an impression on me.

Pi

The circumference of a circle is (π) times the diameter of the circle, where π is approximately $3 \frac{1}{7}$ (or $22/7$ - these sacred numbers turn up frequently in Hindu texts and Rosicrucian literature), or 3.14, or about square root of 10. The value of π is an irrational number, and π to 39 decimal places is sufficient accuracy to calculate the circumference of the universe to within the radius of the hydrogen atom! Here it is to 100 decimal places (see here for 5000 decimal places):

3.141592653589793238462643383279502884197169399375105
8209749445923078164062862089986280348253421170679

*ga - pa - bha - ya - ma - dha - tha - ta | gna - da - sa -
da - cha - ssa - dha - ga*

3 - 1 - 4 - 1 - 5 - 9 - 2 - 6 | 5 - 3 - 5 - 8 - 9 - 7 - 9 - 3

*kha - la - ja - va - ta - kha - ta - va | ga - la - ha -
la - ra - ssa - dha - ra*

2 - 3 - 8 - 4 - 6 - 2 - 6 - 4 | 3 - 3 - 8 - 3 - 2 - 7 - 9 -

2 (? 5 - wrong in the 32nd digit?)

In Vedic Mathematics, Swami Sri Bharati Krsna Tirthaji Majaraja shows the vedic numeric code, and how a hymn in praise of the Lord Shri Krishna, the Lord Shri Shankara, is also the value of π to 32 decimal places (and apparently a key to calculate it to any number of decimal place, which I do not know) - the Sanskrit is shown above, which you can decode in to syllables and the decoding key below.

1 ka, ta, pa, ya

2 kha, tha, pha, ra

3 ga, da, ba, la

4 gha, dha, bha, va

5 gna, na, ma, sa

6 sa, ta, sha

7 cha, tha, ssa

8 ja, da, ha

9 jha, dha

0 ksa

The original sanskrit verse can be found in the appendix of Vedic Mathematics by Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaja.

$$\begin{aligned}
 &= \frac{1}{3} \left[\frac{1}{36} - \frac{1}{14} \right] \\
 &= 0.014550
 \end{aligned}$$

So the displacement of the object from time $t = 2$ to $t = 3$ is 0.015 units.

PROBLEM:

Find:

$$\int_0^1 \sqrt{x^2 + 1} dx$$

We put $u = x^2 + 1$.

So $du = 2x dx$

But the question does not have " $x dx$ " so we cannot solve it using any of the integration methods used above. (It can be done using trigonometric substitution, however).

We need to use numerical approaches. When software like Scientific Notebook, Matlab or Mathcad perform definite integrals, it uses numerical methods.

We use two methods:

- Trapezoidal rule
- Simpson's Rule

We meet these methods in the next 2 sections.

The Trapezoidal Rule

PROBLEM:

Find:

$$\int_0^1 \sqrt{x^2 + 1} dx$$

We put $u = x^2 + 1$ and as $x = 0 \rightarrow 1$, $u = 1 \rightarrow 2$

So $du = 2x dx$

But the question does not contain an $x \, dx$ term so we cannot solve it using any of the normal integration methods.

We need to use numerical approaches. When software like Mathcad or graphics calculators perform definite integrals, they use numerical methods.

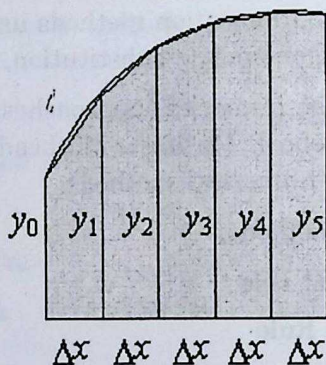
We can use one of two methods:

- Trapezoidal rule
- Simpson's Rule

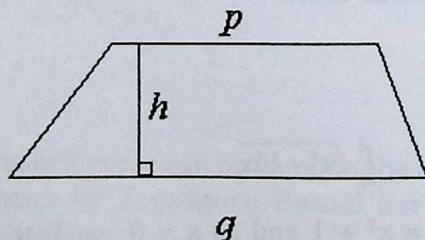
The Trapezoidal Rule

We saw the basic idea in our first attempt at solving the area under the arches problem above.

Instead of using rectangles, we see that trapezoids (trapeziums) give a better approximation to the area.



Now, the area of a trapezoid (trapezium) is given by:



$$\text{Area} = \frac{h}{2}(p+q)$$

So the approximate area under the curve is found by adding the area of the trapezoids. (Our trapezoids are rotated 90° so that their new base is actually the height. So $h = ?x$.)

$$\text{Area} \approx \frac{1}{2}(y_0 + y_1)\Delta x + \frac{1}{2}(y_1 + y_2)\Delta x + \frac{1}{2}(y_2 + y_3)\Delta x \dots$$

We can simplify this to give us the Trapezoidal Rule, for n trapezoids:

$$\text{Area} \approx \Delta x \left(\frac{y_0}{2} + y_1 + y_2 + y_3 + \dots + \frac{y_n}{2} \right)$$

To find Δx for the area from $x = a$ to $x = b$, we use:

$$\Delta x = \frac{b - a}{n}$$

and we also need

$$y_0 = f(a)$$

$$y_1 = f(a + \Delta x)$$

$$y_2 = f(a + 2\Delta x)$$

...

$$y_n = f(b)$$

Note:

- We get a better approximation if we take more trapezoids [up to a limit!].
- The more trapezoids we take, we have: $\Delta x \rightarrow 0$.
- We can write (if the curve is above the x -axis only between $x = a$ and $x = b$):

$$\text{Area} = \int_a^b f(x) dx \approx \Delta x \left(\frac{y_0}{2} + y_1 + \dots + \frac{y_n}{2} \right)$$

EXERCISE:

Using $n = 5$, approximate:

$$\int_0^1 \sqrt{x^2 + 1} \, dx$$

Here, $a = 0$ and $b = 1$.

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{5} = 0.2$$

$$y_0 = f(a) = f(0) = \sqrt{0^2 + 1} = 1$$

$$y_1 = f(a + \Delta x) = f(0.2) = \sqrt{0.2^2 + 1} = 1.0198039$$

$$y_2 = f(a + 2\Delta x) = f(0.4) = \sqrt{0.4^2 + 1} = 1.0770330$$

$$y_3 = f(a + 3\Delta x) = f(0.6) = \sqrt{0.6^2 + 1} = 1.1661904$$

$$y_4 = f(a + 4\Delta x) = f(0.8) = \sqrt{0.8^2 + 1} = 1.2806248$$

$$y_5 = f(b) = f(1) = \sqrt{1^2 + 1} = 1.4142136$$

So the area $\approx$

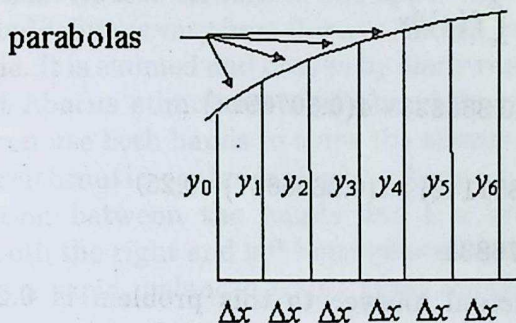
$$\begin{aligned} &0.2 \left(\frac{1}{2} \times 1 + 1.0198039 + 1.0770330 + 1.1661904 \right. \\ &\quad \left. + 1.2806248 \dots + \frac{1}{2} \times 1.4142136 \right) \\ &= 1.150 \end{aligned}$$

$$\text{So } \int_0^1 \sqrt{x^2 + 1} \, dx \approx 1.150$$

Simpson's Rule

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used straight lines to model the curve in trapezoidal Rule.

We seek an even better approximation. In Simpson's Rule, we use parabolas to approximate each part of the curve. This proves to be very efficient.



We can show (by integrating the area under each parabola and adding these areas) that the approximate area is given by Simpson's Rule:

$$\text{Area} = \int_a^b f(x) dx$$

$$\approx \frac{\Delta x^3}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

Note: In Simpson's Rule, n must be EVEN.

In the LiveMath document, note how the approximation is very good, even with a small number of divisions.

EXERCISE:

Approximate $\int_2^3 \frac{dx}{x+1}$ using Simpson's Rule with $n = 4$.

ANSWER

$$\Delta x = (3 - 2)/4 = 0.25$$

$$y_0 = f(a) = f(2) = 1/(2 + 1) = 0.3333333$$

$$y_1 = f(a + \Delta x) = f(2.25) = 1/(2.25+1) = 0.3076923$$

$$y_2 = f(a + 2\Delta x) = f(2.5) = 1/(2.5+1) = 0.2857142$$

$$y_3 = f(a + 3\Delta x) = f(2.75) = 1/(2.75+1) = 0.2666667$$

$$y_4 = f(b) = f(3) = 1/(3+1) = 0.25$$

$$\text{Area} = \int_a^b f(x) dx$$

$$\approx \frac{0.25}{3} (0.333333 + 4(0.3076923)$$

$$+ 2(0.2857142) + 4(0.2666667) + 0.25)$$

$$= 0.2876831$$

[The actual answer to this problem is 0.287682 so our Simpson's Rule approximation has an error of only $3.6 \times 10^{-4}\%$.]

Mental Math Methods from Asia

First of all, let us figure out what exactly is mental math. Today if you search the phrase "mental math" you will probably end up with millions of options. Not exactly that makes your life easy; instead it builds up and strengthens your curiosity. Put in simplest terms, mental math can be defined as calculations performed in your head - mentally - without help of any external device be it as simple as pen and paper or any modern day device such as calculator, computer or any other electronic gadget.

We humans perform mental mathematical calculations everyday, consciously and unconsciously. When you are driving you figure out when to apply brakes to bring the vehicle to stop before hitting something. You figure out time difference between east coast and west coast. But where we falter is at the simplest and most mundane of calculations. Go to a restaurant and figure out 18% gratuity.

Abacus Mental Mathematics

What is abacus mental mathematics? Origin of Abacus is highly disputed today, some say it originated in

Mesopotamia and some claim to be in China. Over centuries, abacus has evolved in to various different forms and sizes. The most commonly used is the Japanese Soroban Abacus.

The Soroban Abacus consists of one upper row and four lower rows and columns vary from thirteen, fifteen, seventeen or twenty one. It is claimed and proven by many researchers in Asia that Abacus stimulates whole brain development. When children use both hands to move the abacus beads to perform arithmetic calculations, there is quick communication between the hands and the brain that stimulates both the right and left hemispheres of the brain. This promotes rapid, balanced whole brain development.

If a child starts learning the abacus before being taught traditional arithmetic, there is minimal conflict and the child will easily work within both systems. If a child starts the program later, having already received traditional foundations, there may be a slightly extended learning period for the child to accept and integrate the abacus method.

Vedic Mental Mathematics

What is Vedic mental mathematics? Origin of Vedic Mathematics is in Atharva Veda (Holy Scripture from Hinduism). Vedic mathematics is a system based on sixteen sutras (aphorisms) which are actually word-formulae describing natural ways of solving a whole range of mathematical problems. These formulae describe the way the mind naturally works and are therefore a great help in directing the student to the appropriate method of solution.

It is claimed and proven by many researchers in Asia that practice and use of Vedic mathematics helps the person in many different aspects of decision making. From intelligent guessing to thinking outside the box ability. Vedic mathematics has its applications to much advanced mathematics, such as calculus and linear algebra. The sixteen sutras are: By one more than the one before, All from 9 and

the last from 10, Vertically and crosswise, Transpose and apply, If the Samuccaya is the same it is zero, If one is in ratio the other is zero, By addition and by subtraction, By the completion or non-completion, Differential calculus, By the deficiency, Specific and general, The remainders by the last digit, The ultimate and twice the penultimate, By one less than the one before, The product of the sum, and All the multipliers

Today, both these methods have made a come back in Asia. Abacus Mental Math method is extremely popular in nations of China, Taiwan, Japan, Malaysia, Korea and India whereas Vedic Mental Math method is extremely popular only in India.

Vedas and Mathematics

Early age mathematicians had made Mathematics a Religion and created a simple set of rules to handle complex problems. This ancient science was lost for a long time only to have been discovered recently.

In this blog I intend to share with you the wisdom behind the sutras written long back by some Brilliant Rishis (learned men). I will try to give as many examples as possible to help you quickly understand the concept and also practice the same.

Please give me sometime to develop the complete material. However you can keep visiting the blog for interesting notes and updates.

To start off, I will try to show how easy it is to solve complex mathematical calculations with the use of a few sutras (rules)...

Example 1: How much time would you take to multiply $67 * 69$? Did you need to use a pen and paper for this?

The traditional way to solve this would be:

61

* 69

 549 ...1 (Multiply 9 to 61)

 366X ...2 (Multiply 6 to 61, with space on the right)

4209 (Sum up the lines 1 and 2)

I could have never done this calculation without pen and paper.

But try this Sutra: BY ONE MORE THAN THE ONE BEFORE

The application of the same:

$$61 * 69 = (6 * 7) + (9 * 1) = 4209$$

Try this now: $47 * 43$:

Check $7+3 = 10$ and the tens digit is same for both numbers being multiplied: So:

$$(4 * 5) + (7 * 3) = 2021$$

Could you reach to the answer before you could type the digits into your computer/calculator?

Interested

Well the question I generally get after this introduction is how many such Sutras (rules) does one have to remember to solve all types of mathematical problems?

The answer is merely 16 Sutras and 13 Sub-Sutras!! These sutras help solve numerous mathematical problems including multiplications, square roots, fractions, HCF, LCM, etc. Compare that to how many tables you had to remember and how many complex procedures you have remembered to solve mathematical problems??

Vedic Mathematics applies to arithmetic, algebra, geometry - plane and solid, trigonometry - plane and

spherical, conics- geometrical and analytical, astronomy, calculus - differential and integral etc., etc.

Vedic Mathematics uses elegant, coherent and unified structures that are direct and easy unlike the methods taught in school. I was afraid of problems due to the rigour involved in doing calculations. Vedic Mathematics opens a fresh way to look at old problems or is it an old way to look at fresh problems!!

Ancient India's Contribution to Mathematics

"India was the motherland of our race and Sanskrit the mother of Europe's languages. India was the mother of our philosophy, of much of our mathematics, of the ideals embodied in Christianity... of self-government and democracy. In many ways, Mother India is the mother of us all." - Will Durant - American Historian 1885-1981

Mathematics represents a high level of abstraction attained by the human mind. In India, mathematics has its roots in Vedic literature which is nearly 4000 years old. Between 1000 BC and 1000 AD, various treatises on mathematics were authored by Indian mathematicians in which were set forth for the first time, the concept of zero, the techniques of algebra and algorithm, square root and cube root.

A Search for Our Present in History

As in the applied sciences like production technology, architecture and shipbuilding, Indians in ancient times also made advances in abstract sciences like Mathematics and Astronomy. It has now been generally accepted that the technique of algebra and the concept of zero originated in India.

But it would be surprising for us to know that even the rudiments of Geometry, called Rekha-Ganita in ancient India,

were formulated and applied in the drafting of Mandalas for architectural purposes. They were also displayed in the geometric patterns used in many temple motifs.

Many motifs in Hindu temples and Palaces display a mix of floral and Geometric patterns.

Even the technique of calculation, called algorithm, which is today widely used in designing soft ware programs (instructions) for computers was also derived from Indian mathematics. In this chapter we shall examine the advances made by Indian mathematicians in ancient times.

Algebra—The Other Mathematics?

In India around the 5th century AD, a system of mathematics that made astronomical calculations easy was developed. In those times its application was limited to astronomy as its pioneers were Astronomers. Astronomical calculations are complex and involve many variables that go into the derivation of unknown quantities. Algebra is a short-hand method of calculation and by this feature it scores over conventional arithmetic.

In ancient India conventional mathematics termed Ganitam was known before the development of algebra. This is borne out by the name - Bijaganitam, which was given to the algebraic form of computation. Bijaganitam means 'the other mathematics' (Bija means 'another' or 'second' and Ganitam means mathematics). The fact that this name was chosen for this system of computation implies that it was recognised as a parallel system of computation, different from the conventional one which was used since the past and was till then the only one. Some have interpreted the term Bija to mean seed, symbolizing origin or beginning. And the inference that Bijaganitam was the original form of computation is derived. Credence is lent to this view by the existence of mathematics in the Vedic literature which was also shorthand method of computation. But whatever the

origin of algebra, it is certain that this technique of computation Originated in India and was current around 1500 years back. Aryabhatta an Indian mathematican who lived in the 5th century AD has referred to Bijaganitam in his treatise on Mathematics, Aryabhattiya. An Indian mathematician - astronomer, Bhaskaracharya has also authored a treatise on this subject, the treatise which is dated around the 12th century AD is entitled 'Siddhanta-Shiromani' of which one section is entitled Bijaganitam.

Thus the technique of algebraic computation was known and was developed in India in earlier times. From the 13th century onwards, India was subject to invasions from the Arabs and other Islamised communities like the Turks and Afghans. Alongwith these invader: came chroniclers and critics like Al-Beruni who studied Indian society and polity.

The Indian system of mathematics could no have escaped their attention. It was also the age of the Islamic Renaissance and the Arabs generally improved upon the arts and sciences that they imbibed from the land they overran during their great Jihad. The system of mathematics they observed in India was adapted by them and given the name 'Al-Jabr' meaning 'the reunion of broken parts'. 'Al' means 'The' & 'Jabr' mean 'reunion'. This name given by the Arabs indicates that they took it from an external source and amalgamated it with their concepts about mathematics.

Between the 10th to 13th centuries, the Christian kingdoms of Europe made numerous attempts to reconquer the birthplace of Jesus Christ from its Mohammedan-Arab rulers. These attempts called the Crusades failed in their military objective, but the contacts they created between oriental and occidental nations resulted in a massive exchange of ideas. The technique of algebr could have passed on to the west at this time.

During the Renaissance in Europe, followed by the industrial revolution, the knowledge received from the east

was further developed. Algebra as we know it today has lost any characteristics that betray its eastern origin save the fact that the term 'algebra' is a corruption of the term 'Al jabr' which the Arabs gave to Bijaganitam. Incidentally the term Bijaganit is still used in India to refer to this subject.

In the year 1816, an Englishman by the name James Taylor translated Bhaskara's Leelavati into English. A second English translation appeared in the following year (1817) by the English astronomer Henry Thomas Colebrooke. Thus the works of this Indian mathematician astronomer were made known to the western world nearly 700 years after he had penned them, although his ideas had already reached the west through the Arabs many centuries earlier.

In the words of the Australian Indologist A.L. Basham (A.L. Basham; *The Wonder That Was India*.) "... the world owes most to India in the realm of mathematics, which was developed in the Gupta period to a stage more advanced than that reached by any other nation of antiquity. The success of Indian mathematics was mainly due to the fact that Indians had a clear conception of the abstract number as distinct from the numerical quantity of objects or spatial extension."

Thus Indians could take their mathematical concepts to an abstract plane and with the aid of a simple numerical notation devise a rudimentary algebra as against the Greeks or the ancient Egyptians who due to their concern with the immediate measurement of physical objects remained confined to Mensuration and Geometry.

Geometry and Algorithm

But even in the area of Geometry, Indian mathematicians had their contribution. There was an area of mathematical applications called *Rekha Ganita* (Line Computation). The *Sulva Sutras*, which literally mean 'Rule of the Chord' give geometrical methods of constructing altars and temples. The

temples layouts were called Mandalas. Some of important works in this field are by Apastamba, Baudhayana, Hiranyakesin, Manava, Varaha and Vadhula.

The Buddhist Pagodas borrowed their plan of construction from the geometric grid of the Mandala used for constructing temples in India (A majestic Pagoda at Bangkok)

The Arab scholar Mohammed Ibn Jubair al Battani studied Indian use of ratios from Retha Ganita and introduced them among the Arab scholars like Al Khwarazmi, Washiya and Abe Mashar who incorporated the newly acquired knowledge of algebra and other branches of Indian mathema into the Arab ideas about the subject.

The chief exponent of this Indo-Arab amalgam in mathematics was Al Khwarazmi who evolved a technique of calculation from Indian sources. This technique which was named by westerners after Al Khwarazmi as "Algorismi" gave us the modern term Algorithm, which is used in computer software.

Algorithm which is a process of calculation based on decimal notation numbers. This method was deduced by Khwarazmi from the Indian techniques geometric computation which he had stied. Al Khwarazmi's work was translated into Latin under the title "De Numero Indico" which means 'of Indian Numerals' thus betraying its Indian origin. This translation which belong to the 12th century A.D credited to one Adelard who lived in a town called Bath in Britian.

Thus Al Khwarazmi and Adelard could looked upon as pioneers who transmit Indian numerals to the west. Incidents according to the Oxford Dictionary, word algorithm which we use in the English language is a corruption of the name Khwarazmi which literally means '(a person) from Khawarizm', which was the name of the town where Al Khwarazmi lived. To day unfortunately', the original Indian texts that Al Khwarazmi studied arelost to us, only the

translations are available. The Arabs borrowed so much from India the field of mathematics that even the subject of mathematics in Arabic came to be known as *Hindsa* which means 'from India' and a mathematician or engineer in Arabic is called *Muhandis* which means 'an expert in Mathematics'. The word *Muhandis* possibly derived from the Arabic term mathematics viz. *Hindsa*.

The Concept of Zero

The concept of zero also originated in ancient India. This concept may seem to be a very ordinary one and a claim to its discovery may be viewed as queer. But if one gives a hard thought to this concept it would be seen that zero is not just a numeral. Apart from being a numeral, it is also a concept, and a fundamental one at that. It is fundamental because, terms to identify visible or perceptible objects do not require much ingenuity.

But a concept and symbol that connotes nullity represents a qualitative advancement of the human capacity of abstraction. In absence of a concept of zero there could have been only positive numerals in computation, the inclusion of zero in mathematics opened up a new dimension of negative numerals and gave a cut off point and a standard in the measurability of qualities whose extremes are as yet unknown to human beings, such as temperature.

In ancient India this numeral was used in computation, it was indicated by a dot and was termed *Pujyam*. Even today we use this term for zero along with the more current term *Shunyam* meaning a blank. But queerly the term *Pujyam* also means holy. *Param-Pujya* is a prefix used in written communication with elders. In this case it means respected or esteemed. The reason why the term *Pujya* - meaning blank - came to be sanctified can only be guessed.

Indian philosophy has glorified concepts like the material world being an illusion *Maya*), the act of renouncing the

material world (Tyaga) and the goal of merging into the void of eternity (Nirvana). Herein could lie the reason how the mathematical concept of zero got a philosophical connotation of reverence.

In a queer way the concept of 'Zero' or Shunya is derived from the concept of a void. The concept of void existed in Hindu Philosophy hence the derivation of a symbol for it. The concept of Shunyata, influenced Southeast Asian culture through the Buddhist concept of Nirvana 'attaining salvation by merging into the void of eternity' (Ornate Entrance of a Buddhist temple in Laos).

It is possible that like the technique of algebra; the concept of zero also reached the west through the Arabs. In ancient India the terms used to describe zero included Pujiyam, Shunyam, Bindu the concept of a void or blank was termed as Shukla and Shubra. The Arabs refer to the zero as Siphra or Sifr from which we have the English terms Cipher or Cypher. In English the term Cipher connotes zero or any Arabic numeral. Thus it is evident that the term Cipher is derived from the Arabic Sifr which in turn is quite close to the Sanskrit term Shubra.

The ancient India astronomer Brahmagupta is credited with having put forth the concept of zero for the first time: Brahmagupta is said to have been born the year 598 AD at Bhillamala (today's Bhinmal) in Gujarat, Western India.] much is known about Brahmagupta's early life. We are told that his name as a mathematician was well established when K Vyaghramukha of the Chapa dyansty m him the court astronomer. Of his two treatises, Brahma-sputa siddhanta and Karanakhandakhadyaka, first is more famous. It was a corrected version of the old Astronomical text, Brahma siddhanta. It was in his Brahma-sphu siddhanta, for the first time ever had be formulated the rules of the operation zero, foreshadowing the decimal system numeration. With the integration of zero into the numerals

Magic Squares

Translating Numbers Into Art 6 1 8 7 5 3 2 9 4 Imagine in front of you some playing cards and the ancient puzzle was to create a 3×3 square such that all columns, rows and diagonals that have the same sum. As shown above, the Magic Sum, aka the Magic Square Constant = 15. There is a treasure house of knowledge encoded within this harmonic matrix that the ancients knew and hid. Magic Squares for thousands of years have fascinated many cultures and many great minds like Benjamin Franklin and Albrecht Durer. Why was it at the centre of the Tibetan Calendar/Cosmolgy (Time Code / Applications in Time Travel)? Why did Ambaji, of the Hindu pantheon, the ancient goddesses of Creation (the universe flows through her breast), have as her tantric symbol, this Magic Square of 3×3 , which has 8 possible permutations? and which Buddha, in his enlightenment, described in detail the 8 sided nature of sub-atomic molecules: the "Acta-Kalapos". Durga and Kali and Shakti etc. have variations of the same.

Why is it that when you translate NUMBERS INTO ART, by drawing a long connecting line, from 1 to 2 to 3 etc. to the last number, that elegant and phi-ratioed patterns appear? that when tiled or tessellated, produce the atomic structure of Diamond (which is verified by the clairvoyant drawings of the Occult Chemists: Charles Leadbeater and Annie Besant. Why is it that the pattern for the Magic Square of 11×11 is almost identical to the Ley Line Grid developed by Bruce Cathy, a New Zealand pilot, which was originally the jig-saw-puzzle-map of UFO sightings in his area? Why did Professor Searl, of the UK, have his work on levity discs and over-unity generators banned?

The metals of his generators for the space-craft he built were intimately related, by proportion, to the atomic table of elements and to the opposing pairs of the above Magic Square of 3×3 , eg: notice that all pairs around the central cell have a sum of 10, thus 2 parts gold was alloyed with

8 parts silver, or 3 parts titanium to 7 parts nickel. Why did King Solomon the Rabbi, design his Seal based on the 3 rows of the above Magic Square being translated into a Natural Square of 3 (that has its numbers in Counting Order)? In this day lesson, you will superimpose a Magic Square of 7×7 , by rotation of 90 degrees upon itself, to reveal ancient symbols at its centre: the Hindu Peace Symbol of the Swastika nesting within the Christian Cross!

The ancient Essenes adored the richness of symbolism within these Mystic Squares and gave accounts or Parables of Jesus (Isa or Iesu) that related to our sense of community symbolized by the magic square, that each person, so positioned, has a divine role to play, that if one number was taken away or moved, it would no longer be magical, to realise 'Order amidst the Chaos', and the ancient axiom: 'As Above.. so below'.

Some questions to ponder upon:

- * How could the Aryan civilisation, near the Hunza Valley, 3,000 years ago, mentally compute mathematical operations that today only a calculator could achieve?
- * And if the maths could not be done mentally, how could they do everything we know today, IN ONE-LINE? eg: converting the fraction $1/19$ into a decimal is an 18 digit answer that requires 35 lines of exhausting long division, how could they do it in a one simple line solution?
- * Why is NASA secretly using this MENTAL ONE-LINE SYSTEM OF ARITHMETIC in the field of Advanced Artificial Intelligence? and not incorporating it into our current mathematics curriculum?
- * Why are many of Bill Gate's Mathematicians Indians?
- * Is the decimal form of Pi a sonic code for dedications to Krishna?

- * What does the Bhagavad Gita have to say about this lost science of Clairvoyant Calculation that the ancient Temple Builders employed?
- * How can you become a Human Bio-Calculator?
- * Is the current Western Maths Curriculum keeping our Children in a state of limited intelligence? The answer to all these questions are found by understanding the ancient system of Vedic Mathematics as rediscovered by Bharati Krsna Tirthaji (1884- 1960), of Puri Govardhan Math, dux of India in Mathematics and a Sanskrit Philosopher who wrote and spoke perfect English, he reached the position of a Shankacharya (the spiritual head of India 100 years ago, like a Dalai Lama or Vatican Pope), renunciated several high official positions in society to retreat into a forest for 7 years as a yogi, between 1911 and 1918 in which time he collected all the lost mathematical knowledge (Ganita Sutras), improved and reconstructed them to 16 basic Sutras or word formulae that solve every known mathematical problem.

These 16 Volumes relating to the 16 Sutras was our legacy from the ancient past. This highly intelligent and sophisticated system of Mental, One-Line Arithmetic was taught by The Master to his main student, who today is the leading exponent in the world: Maharishi Mahesh Yogi, the long white-haired and bearded yogi that many of the 60s generation know of as the Beatle's guru. He is considered as the greatest human bio-calculator on the planet today, holding the vision of raising the collective consciousness of all nations by re-introducing the lost Vedic Mathematics Sutras early in a child's spiritual and educational development. Imagine a maths class in India today, like in the times of old when knowledge was passed on as an oral tradition, and the teacher quickly calls or drills out certain

numbers to multiply or divide and the children instantly reply with the correct intuitive answers!

Today, in schools where Maharishi's Transcendental Meditation technique are used students perform remarkably well academically, viewing Mathematics as a Universal Language and therefore often winning the highest competitive prizes in mathematics. Maharishi organised the scattered Vedic Literature into a complete science of consciousness for both its theoretical and practical value, describing the 16 Sutras as fundamental impulses of intelligence that structure all the Laws of Nature and those that govern the Universe. Maharishi, the Founder of TM, once said: "Vedic Mathematics is that one field of knowledge which fulfils the purpose of education by developing the total creative genius of the individual, giving him/her the ability to be always spontaneously right and automatically precise so that his/her action, supported by Natural Law, is always effortlessly fulfilling."

Unfortunately, in 1958 after Bharati Krsna Tirthaji returned from the USA tour, sponsored by Paramahansa Yogananda (who introduced Yoga to the West and created the Self-Realisation Foundation) all of Bharati Krsna's 16 Volumes for the 16 Sutras were confirmed lost. Disciples observed that people in the streets of India were imitating Tirthaji's lecture's and dazzling audiences with mathematically mis-used psychic powers, and begged the Master to write a book just before his death in 1960. This single volume, called VEDIC MATHEMATICS, has become the bible on this subject and spawned a thousand web-sites worldwide. Maharishi schools disseminated the teachings to England where certain schools have adopted the Vedic curriculum but the irony is that today India is struggling to release the British curriculum and NASA have adopted it fully in the realms of advanced robotics.

Calculations that can be solved as quick as lightning are a great tool to adopt, but you would not want to teach it

worldwide in the fear that you may churn out a generation of child geniuses that may threaten the intellectual status quo. The gift that the Hindus gave to world, thousands of years ago, and that which is currently responsible for a global silicon chip technology, was none other than the invention of ZERO and the use of the decimal point. We call our common numbers "Arabic Numerals" but really they extend back to the Hindu concept of creation and void known as "Bindu" or The Zero Point. In Maths, it makes the magic work eg: if you wish to multiply 98×98 we need to understand Zero and Unity.

All Vedic Maths is based on the understanding of Unity Consciousness which means they utilise processes or Number Bases that correspond to: 0, 10, 100, 1000, 10000 etc. all of which add to 1. Let us look at some practical examples, in light of the fact that the Vedas, literally "the illimitable storehouse of All Knowledge" came under 4 headings or categories like Ayurveda (Healing), and Vedas for Music and Astronomy etc. and one relevant sub-Veda that related to Temple Building and Engineering and therefore Mathematics called SthapathyVeda.

Thus a Vedic Mathematician was also an astronomer, healer and poet. It was a total system. As a temple builder, there was no pen and paper, you simply calculated in your head. You are out in the field and you need to tile a floor that is, say 98 units square. How do you do it with mental ease? The Squaring of Numbers near a Base To solve 98 Squared (98×98) we must first determine what Base we are in. It is close to 100, therefore we say Base 100.

We must now choose one of the 16 Sutras to effectively solve the problem. It is called: "By The Deficiency": "By whatever the deficiency, lessen it further by that much and set up the square thereof" says one of the 16 Sutras. Sounds cryptic and meaningless yet it quickly solves the problem.

We get our answer by merely knowing how much is 100 less 98. Knowing that the deficiency is 2, we merely lessen

98 by 2 and then we tag on the squaring of that 2. As a one-line answer, the setting out would appear as thus:

98 Squared = $98 - 2/2 \times 2$. Simplifying it: = $96 / _ 4$ We almost have our answer.

What we need to know is that since our Base is 100, it has 2 zeroes, therefore this fact governs the need for 2 spaces or 2 digits after the "forward slash" symbol (/). By inserting or inventing The Zero as a "Place Marker", the answer is achieved: $98 \text{ Squared} = 96 / 04 = 9,604$. (Perhaps the greatest invention of all time, the Hindu creation of the Zero was to change the world unimaginably as time passed on leading to the ability to send Vimana U.F.O craft and rockets into space, to supercomputers and thus the ancient and continuing battle between Michael and Lucifer: Archangel Michael's Internal Light Vehicle Merkabah Time/Space Travel and Natural Spiritual Powers Versus Lucifer's daring invention of the External Merkabah Technological Metal Spacecraft).

Observe similar examples:

$$97 \text{ Squared} = 97 - 3 / 3 \times 3 = 94 / 09$$

$$96 \text{ Squared} = 96 - 4 / 4 \times 4 = 92 / 16.$$

When the number being squared is above the base, of 100 here, we add the Excess and Square the Excess:

$$104 \text{ Squared} = 104 + 4 / 4 \times 4 = 108 / 16 = 10, 816$$

$$104 \times 105 = 104 + 5 / 4 \times 5 = 109 / 20 = 10, 920$$

What if we enlarged our numbers to 998 Squared?

It is close to 1,000 so we say Base 1,000 and know to have 3 zeroes on the right hand side of the (/).

$$998 \text{ Squared} = 998 - 2 / 2 \times 2 = 996 / _ _ 4 = 996 / 004. \\ = 996,004$$

Understanding this, you can be calculating digits in the millions: $9998 \text{ Squared} = 9998 - 2 / 2 \times 2 = 9996 / _ _ _ 4$

(Since we are in Base 10,000 the 4 Zeroes determine the need for 4 digits after the (/). = $9996 / 0004 = 99,960,004$.)

There is a world-wide debate currently raging about the efficacy of Vedic Mathematics versus the crumbling foundations of Western Mathematics. Generally speaking, the theorems we all learned at school are not wrong but clumsy.

Some of the Western geometrical formulae are certainly wrong or inadequate: for example, the formulae for sphere packing in the higher dimensions increase up to the 6th Dimension then suddenly decrease for higher dimensions, which is simply absurd.

Unfortunately, some die-hard senior mathematicians in an attempt to protect the crumbling foundations that they now stand on feel threatened by the lightning quick mental calculations of the Vedic seers, and go to great lengths to deride Vedic maths as a "bag of tricks". Here is another example illustrating its utter simplicity: "the Path of Least Resistance".

Sutra: Vilokanam, Means by Mere Intuition

Perhaps the greatest of all Sutras is the one known as Vilokanam which literally means: "By Mere Inspection" or "By Mere Observation" or simply "By Mere Intuition".

By relying on modern electronic calculators for the next 20 years or so, without cultivating mental calculations which build the "Mental Muscle", will sadly produce a generation of young adults whose brain will have undoubtedly atrophied! Vedic Mathematics promotes clarity and sharpness of mind as well as improving memory skills and self-confidence, by awakening the Pineal Gland (or Third Eye) to clairvoyantly "see" or intuitively know the predicted answer.

Mystics say that the Pineal gland, located in the exact centre of the brain, was once the size of large cherry, but now it is the size of a pea. Two reasons are that we drink denatured defunct water and we no longer build the Mental Muscle, over-relying on calculators etc. to solve simple

problems. The brain is a muscle, being 90% water, it needs to be hydrated with pure alive waters and regularly exercised. By application of the key Sutra called Vilokanam which means: "By Mere Inspection", a child can know for example that the Cube Root of 35,937 without a calculator, to be equal to 33. By Mere Observation! No working out, no pen or paper.

Mathematics without Tears. "On seeing this kind of work actually being performed by the little children, the doctors, professors and other "big guns" of mathematics are wonder struck and exclaim:- "Is this mathematics or magic? And we invariably answer and say: "It is both. It is magic until you understand it; and it is mathematics thereafter"...

" *Bharati Krsna Tithaji (1884-1960)*

Discoverer of Vedic Mathematics.

"WATCHING A CHILD GO THROUGH OUR EDUCATION SYSTEM IS LIKE WATCHING REVERSE METAMORPHOSIS: IN FLIES A BEAUTIFUL BUTTERFLY, OUT CRAWLS A CATERPILLAR."

If you disliked Mathematics in your school years, do not be disalarmed. It is actually a blessing as your Higher Self or Intelligence refused to embrace a system that was anything less than pure Star Language. If you observe Fig 3, relating to the Sutra "Vertically and Crosswise" for multiplication of 2 Digits by 2 Digits or 3 Digits by Digits, etc. the pictures actually look like star glyphs or codes. The Brain thinks in symbols and pictures. Maths must be taught laden with Shapes, Colour, Aromatherapy and Music for effective Whole-Brain Learning.

Vedic Numerical Codified Knowledge

How the ancient seers sang the long decimal form of Pi. In ancient India, they would sing songs to memorise long

decimals. The Brahmins or scholarly caste, secretive of their knowledge, sonically encrypted mathematical formulae into their devotional praises or hymns to Lord Shri Krishna and also recorded historical data in codified lyrics. Similar to numerology, by ascribing the values of numbers to consonant letters as in A = 1, B = 2, C = 3, D = 4, but in Sanskrit, the Vedic Numerical Code was so sophisticated that it possessed 3 layers and therefore triple meanings.

It turns out that decimal form of the transcendental number: $\text{Pi} = 3.1415926535897932384626433832792$ etc. was hidden or codified in these following syllables, by chanting: "Gopi Bhagyamaduv rata Shringishodadi Sandiga Kala Jeevitarava Tava Galaddhalara Sangara" (Observing the top line, "go" = 3, "pi" = 1, "bha" = 4, "ya" = 1, "ma" = 5, "dhu" = 9, "ra" = 2, "ta" = 6 etc. giving the first 8 figures of Pi, the ratio of the circumference of a circle to its diameter). Not only did it give Pi to 32 decimal places, but there was a secret Master Key within the patterning of the 32 that could unlock the next 32 decimals of Pi, and so on, a ticket to infinity. Not only did it praise Krishna, it operated on another level as a dedication to Shankara: (800A.D, the celebrated Acharya or Teacher, and founder of 4 monastic orders.

He could read at the age of 2 and mastered the Vedas at the age of 8. He wrote scholarly commentaries on the Bhagavad-Gita and the Upanishads which led to the decline of Buddhism in India. He is considered as a partial incarnation of Shiva the Avatara). To conclude this article, the time has come that all past secret knowledge can no longer be kept secret. The Brahmins, by keeping this knowledge complex and inaccessible to the masses, and therefore not shareable, did not live in Truth. The Truth is that we have entered an Age where if something is not Simple and Shareable, then it has no value. It is therefore ripe time to reintroduce the ancient seers infallible Mental and One-Line System of Vedic Mathematics.

Topics Include:

- * Magic Squares and their Atomic Artforms.
- * The Vedic Square and it's Digital Sums.
- * The Divine PHI Proportion or The Living Mathematics of Nature,
- * The 5 Platonic Solids and The 13 Archimedean Solids
- * Pythagorean Mystery School.
- * Vedic Mathematics. He works with Gifted and Talented children and lectures at Maths Camps, Universities and Maths Conferences. He is currently writing a series of 4 books for the Australian Curriculum on Vedic Mathematics. As an Authorised School Performer of a Travelling Show called "Mathemagics", Jain has taught thousands of children and adults in Australia, the wonder of the pure mathematical principles relating to Atomic Art and Discovery. In 1993 this travelling Sacred Geometry Show was banned in Queensland schools, loved by mathematicians but shunned by the top officials in the Department of Education, who are Fundamental Christian-based and antagonistic towards the New Maths.

Bibliography

- A B Chace, L S Bull, H P Manning and R C Archibald: *The Rhind Mathematical Papyrus*, Oberlin, Ohio, 1927.
- A.L. Basham: *The Wonder That Was India*, Rupa & Co., Calcutta, 1967.
- Aaboe, Asger: *Episodes from the Early History of Mathematics*, New York, 1964.
- Amma, Saraswari: *Geometry in Ancient and Medieval India*, Motilal Banarsidas, 1979.
- Armstrong, Robert G.: *Yoruba Numerals*, London, Oxford University Press, 1962.
- Arnett, William S.: *The Predynastic Origin of Egyptian Hieroglyphs*, Washington, University Press of America, 1982.
- Ascher, Marcia: *Ethnomathematics*, Pacific Grove, CA: Brooks/Cole, 1991.
- Bag, A. K.: *Mathematics in Ancient and Medieval India*, Varanasi: Chaukhambha Orientalia, 1979.
- Barnard, F. P.: *The Casting-Counter and the Counting Board. A Chapter in the History of Numismatics and Early Arithmetic*, Oxford, Oxford University Press, 1916.
- Barrow, John D.: *Pi in the Sky*, Oxford, Clarendon Press, 1992.
- Basham, A. L.: *The Wonder That Was India*, Calcutta, Rupa & Co., 1967.
- Beckmann, Petr: *A History of Pi*, New York, St. Martin's Press, 1974.
- Bell, Eric T.: *The Magic of Numbers*, New York, McGraw-Hill, 1946.

- Berlin, Brent: *Tzeltal Numeral Classifiers*, The Hague, Mouton, 1968.
- Bharati Krishna Tirthaji Majaraja.: *Vedic Mathematics*. Delhi, Motilal Banarsidass, 1988.
- Bidyaranya, Swami: *History of Hindu Mathematics*, Bombay, Asia Pub. House, 1962.
- Bischoff, Bernard: *Latin Palaeography*, Cambridge, Cambridge University Press, 1990.
- Bonfante, Larissa: *Etruscan*. Berkeley, University of California Press, 1990.
- Bowditch, Charles P.: *The Numeration, Calendar Systems and Astronomical knowledge of the Mayas*, Cambridge, Cambridge University Press, 1910.
- Brainerd, Charles J.: *The Origins of the Number Concept*, New York, Praeger, 1979.
- Brunschwig, Jacques and Geoffrey E.R. Lloyd.: *Greek Thought: A Guide to Classical Knowledge*, Cambridge, Harvard University Press, 2000.
- Bunt, L. N. H., Jones, P. S., & Bedient, J. D.: *The Historical Roots of Elementary Mathematics*, New York, Dover, 1988.
- Chattopadhyaya, D. P.: *Mathematics, Astronomy, and Biology in Indian Tradition: Some Conceptual Preliminaries*, New Delhi, Project of History of Indian Science, Philosophy, and Culture, 1995.
- Clawson, Calvin: *The Mathematical Traveller*, New York, Plenum Press, 1994.
- Closs, Michael P.: *Native American Mathematics*, Austin, University of Texas Press, 1986.
- Conant, Levi L.: *The Number Concept*, New York, MacMillan, 1896.
- Crossley, John N: *The Emergence of Number*, Singapore, World Scientific, 1987.
- Crump, Thomas: *The Anthropology of Numbers*, Cambridge, Cambridge University Press, 1990.

- Dantzig, Tobias: *Number, the Language of Science*, London, George Allen and Unwin, 1940.
- De Villiers, Melius: *The Numeral-Words: Their Origin, Meaning, History and Lesson*, London, Witherby, 1923.
- Dehaene, Stanislas: *The Number Sense*, New York, Oxford University Press, 1997.
- Dilke, Oswald Ashton Wentworth: *Mathematics and Measurement*, Berkeley, CA: U. of Cal. Press, 1987.
- Eves, Howard: *An Introduction to the History of Mathematics*, New York, Rinehart and Company Inc., 1953.
- Flegg, Graham: *Numbers: Their History and Meaning*, London, André Deutsch, 1983.
- Fowler, David: *The Mathematics of Plato's Academy*, Oxford, Clarendon, 1999.
- Fuson, K.C.: *Children's Counting and Concepts of Number*, New York, Springer-Verlag, 1988.
- Ganeri, Anita: *The Story of Numbers and Counting*, New York, Oxford University Press, 1996.
- Gokhale, Shebhana Laxman: *Indian Numerals*, Poona, Deccan College, 1966.
- Griffith, Francis Llewellyn: *Hieratic Papyri from Kahun and Gurob*, London, Bernard Quaritch, 1898.
- Grover, Sudarshan: *History of Development of Mathematics in India*, Delhi, Atma Ram & Sons, 1994.
- Haldane, John Burdon Sanderson: *Science and Indian Culture*, Calcutta, New Age Publishers, 1965.
- Halliwell, James Orchard: *Rara Mathematica*, London, J.W. Parker, 1839.
- Herbert Meschkowski: *Ways of Thought of Great Mathematicians*, Holden-Day Inc., San Francisco, 1964.
- Hogben, Lancelot: *Mathematics for the Million*, New York, W. W. Norton, 1937.
- Howard Eves: *An Introduction to the History of Mathematics*, Rinehart and Company Inc., New York, 1953.

- Ifrah, Georges: *From One to Zero: A Universal History of Numbers*, New York, Viking Penguin, 1985.
- Jagadguru Swami Shri Bharati Krishna Tirthaji Maharaja: *Vedic Mathematics*, Motilal Banarsidass, Delhi, 1988.
- Jaggi, O. P.: *Science and Technology in Medieval India*, New Delhi, Atma Ram & Sons, 1977.
- Kline, Morris: *Mathematical Thought from Ancient to Modern Times*, New York, Oxford U. Press, 1990.
- Kuppuram, G & K. Kumudamani: *History of Science and Technology in India*, Delhi, Sundeep Prakashan, 1990.
- Lancy, David F.: *Cross-Cultural Studies in Cognition and Mathematics*, New York, Academic, 1983.
- Marcus, Joyce: *Mesoamerican Writing Systems*, Princeton, Princeton University Press, 1992.
- Marshack, Alexander: *The Roots of Civilization*, New York, McGraw-Hill, 1972.
- Martzloff, Jean-Claude: *A History of Chinese Mathematics*, New York, Springer, 1997.
- McLeish, John: *The Story of Numbers*, New York, Fawcett Columbine, 1991.
- Menninger, Karl: *Number-Words and Number-Symbols*. London, MIT, 1969.
- Murthy, T. S. Bhanu: *A Modern Introduction to Ancient Indian Mathematics*, Wiley Eastern Ltd., New Delhi, 1992.
- Neugebauer, Otto and A. Sachs: *Mathematical Cuneiform Texts*, New Haven, Yale University Press, 1945.
- Neugebauer, Otto: *The Exact Sciences in Antiquity*, Princeton, Princeton University Press, 1952.
- North, John: *The Norton History of Astronomy and Cosmology*, New York, W.W.Norton, 1995.
- Pandit, M. D.: *Mathematics as known to the Vedic Samhitas*, Delhi, Sri Satguru Publications, 1993.

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- Pappas, Theoni: *The Magic of Mathematics*, San Carlos, Wide World Publishing / Tetra, 1994.
- Parpola, Asko: *Deciphering the Indus Script*, Cambridge, Cambridge University Press, 1994.
- Peet, T. Eric: *The Rhind Mathematical Papyrus*, London, British Museum, 1923.
- Piaget, Jean: *The Child's Conception of Number*, New York, W.W. Norton, 1965.
- Pullan, J.M: *The History of the Abacus*, New York, Praeger, 1968.
- Rao, S. Balaachandra: *Indian Mathematics and Astronomy*, Bangalore, Jnana Deep Publications, 1994.
- Restivo, Sal: *The Social Relations of Physics, Mysticism and Mathematics*, Dordrecht, Kluwer, 1983.
- Robins, G. and C. Shute: *The Rhind Mathematical Papyrus*, London, British Museum, 1987.
- Russell, Bertrand: *The Principles of Mathematics*, Cambridge, Cambridge University Press, 1903.
- Salomon, Richard: *Indian Epigraphy*, New York, Oxford University Press, 1998.
- Saraswati Amma: *Geometry in Ancient and Medieval India*, Motilal Banarsidas, 1979.
- Sarton, George: *The Study of the History of Mathematics*, Cambridge, Cambridge University Press, 1936.
- Satterthwaite, Linton, Jr.: *Concepts and Structures of Maya Calendrical Arithmetics*, Philadelphia, University of Pennsylvania, 1947.
- Schaaf, W.L.: *Mathematics, Our Great Heritage*, New York, Harper and Row, 1948.
- Schimmel, Annemarie: *The Mystery of Numbers*, New York, Oxford University Press, 1993.
- Scriba, C.J.: *The Concept of Number*, Zurich, Mannheim, 1968.

- Seidenberg, A.: *The Zero in the Mayan Numerical Notation*, Austin, University of Texas Press, 1986.
- Smeltzer, Donald: *Man and Number*, New York, Emerson, 1959.
- Smith, D. E.: *History of Mathematics*, New York, Dover Publications Inc., 1951.
- Srinivasiengar, C. N.: *The History of Ancient Indian Mathematics*. Calcutta, World Press, 1967.
- Struik, Dirk J.: *A Concise History of Mathematics*, New York, Dover, 1987.
- Thompson, Sir J. Eric S.: *Maya Hieroglyphic Writing: An Introduction*, Norman, University of Oklahoma Press, 1971.
- Todd, R.R., P.J. Barber, and D. Jones: *The Internal Representation of Number: Analogue or Digital?*, Oxford, Clarendon, 1987.
- Urton, Gary: *The Social Life of Numbers*, Austin, University of Texas Press, 1997.
- Volodarsky, A.I.: *Mathematics in Ancient India*, Tokyo, Science Council of Japan, 1974.
- Woolner, Alfred C.: *Introduction to Prakrit*, Calcutta, Baptist Mission Press, 1917.

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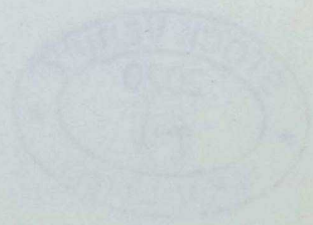
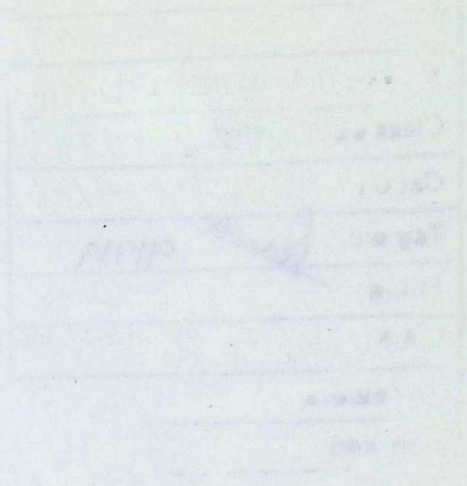
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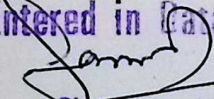
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